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DETERMINATION OF α_s & THE CORRELATED REPLICA METHOD

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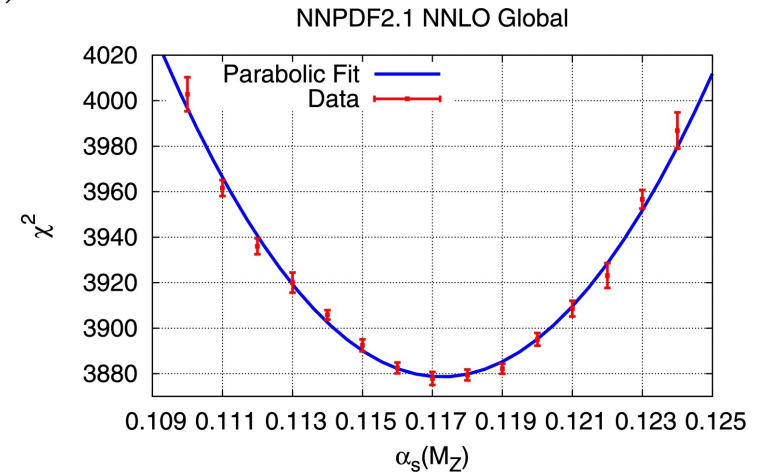
PDF4LHC

CERN, MARCH 28, 2018

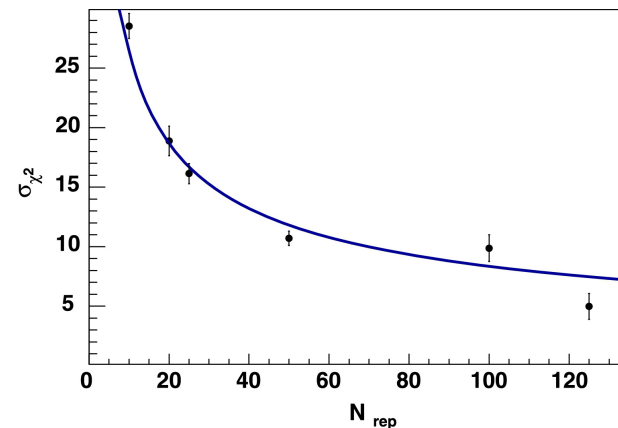
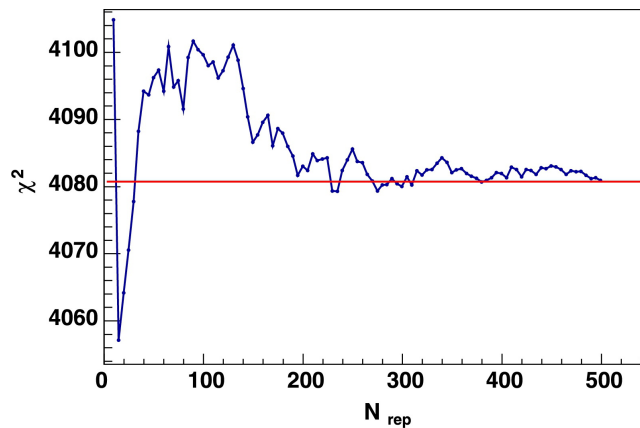
α_s FROM REPLICAS

NNPDF2.1 (2012)

- PERFORM GLOBAL FIT WITH DIFFERENT FIXED VALUES OF α_s
- DETERMINE χ^2 & FIT PARABOLA TO PROFILE
- BEST-FIT FROM MINIMUM UNCERTAINTY FROM $\Delta\chi^2 = 1$

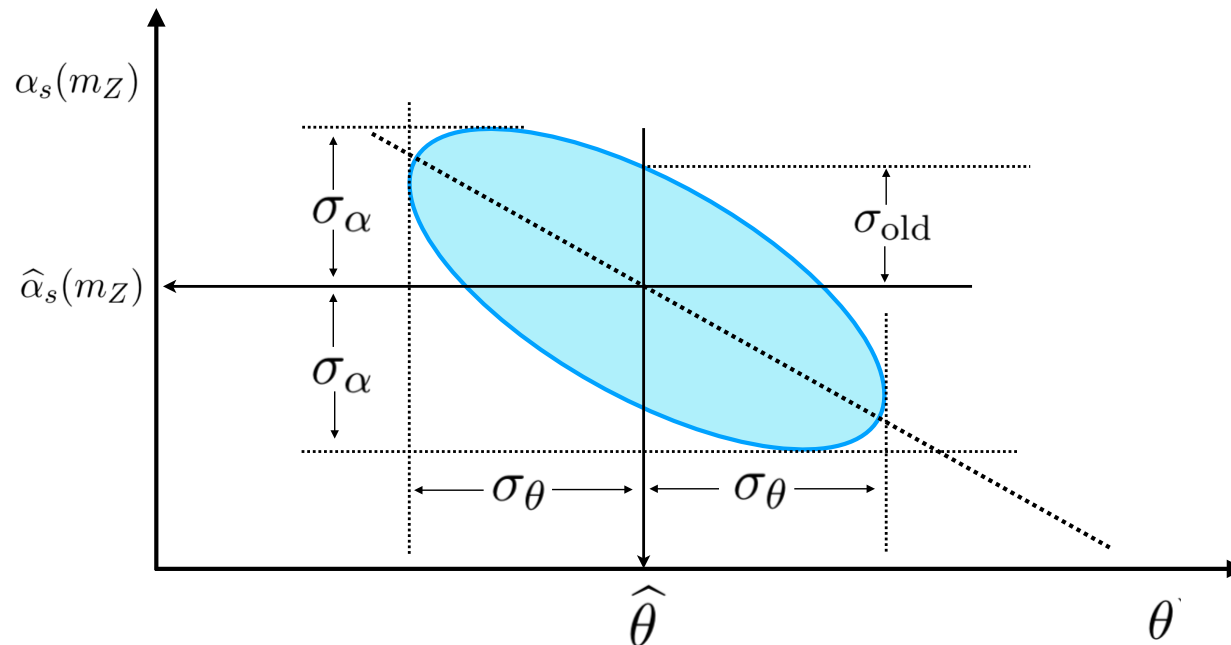


PROBLEM (TRIVIAL)



- $\sigma_{\chi^2} = \sqrt{2N_{\text{dat}}} \Rightarrow$ NEED LARGE N_{rep} TO SUPPRESS χ^2 FLUCTUATIONS

α_s FROM REPLICAS
NNPDF2.1 (2012)
PROBLEMS (LESS TRIVIAL)



- MINIMUM DETERMINED ALONG THE “BEST PDF” LINE $\Rightarrow \sigma_{\text{old}}$
FOR HIGHLY CORRELATED VARIABLES & UNEQUAL SEMIAXES,
MAY UNDERESTIMATE ONE- σ ERROR $\Rightarrow \sigma_\alpha$
- IN NNPDF METHODOLOGY,
PDF UNCERTAINTY $\Leftrightarrow$ DETERMINED FROM REPLICA SAMPLE VARIANCE
 $\Rightarrow$ IS IT $\Delta\chi^2 = 1$? (TOLERANCE?)

NEED SIMULTANEOUS MINIMIZATION IN (PDF, α_s) SPACE!

THE CORRELATED REPLICA METHOD

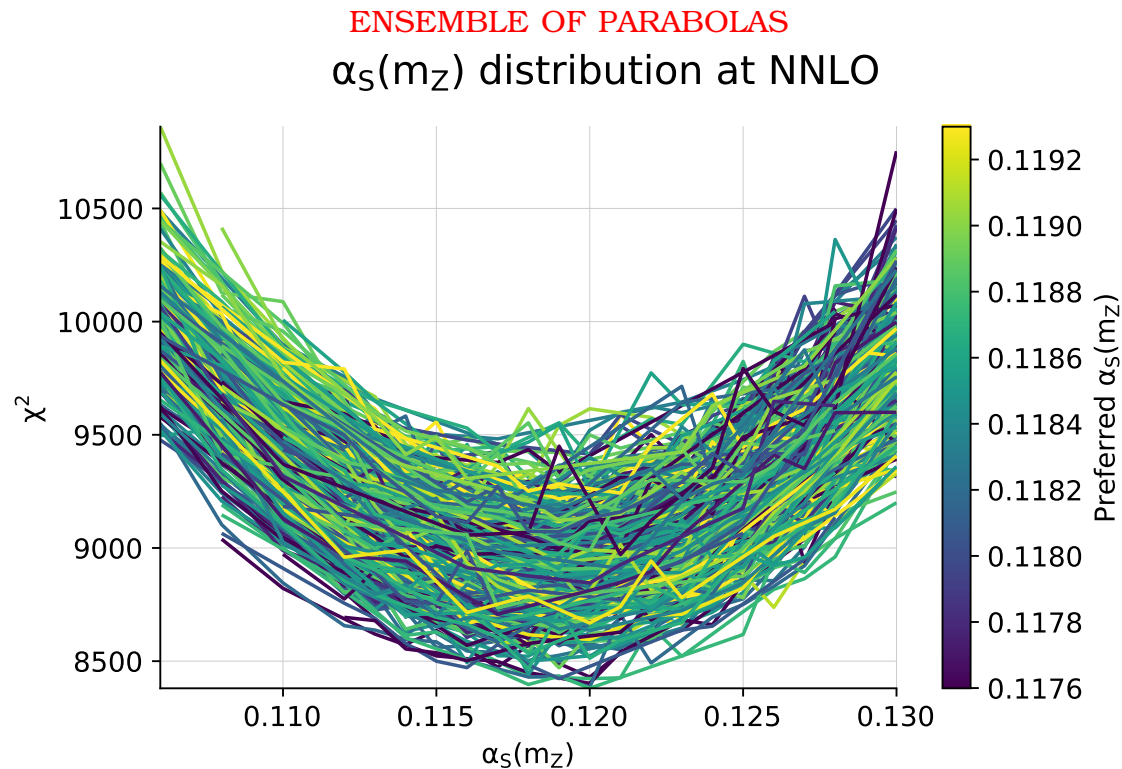
NNPDF3.1 (2018)

- NNPDF METHOD $\Rightarrow$ EACH PDF REPLICA FITTED BY GA TO DATA REPLICA
- IDEALLY PERFORM GENETIC MINIMIZATION IN (PDF, α_s) SPACE
- PROBLEM THEORY PREDICTION $\Leftrightarrow$ PRECOMPUTED GRIDS
DEPEND ON $\alpha_s \Rightarrow$ DIFFICULT TO TREAT AS CONTINUOUS PARAMETER

THE CORRELATED REPLICAS METHOD

NNPDF3.1 (2018)

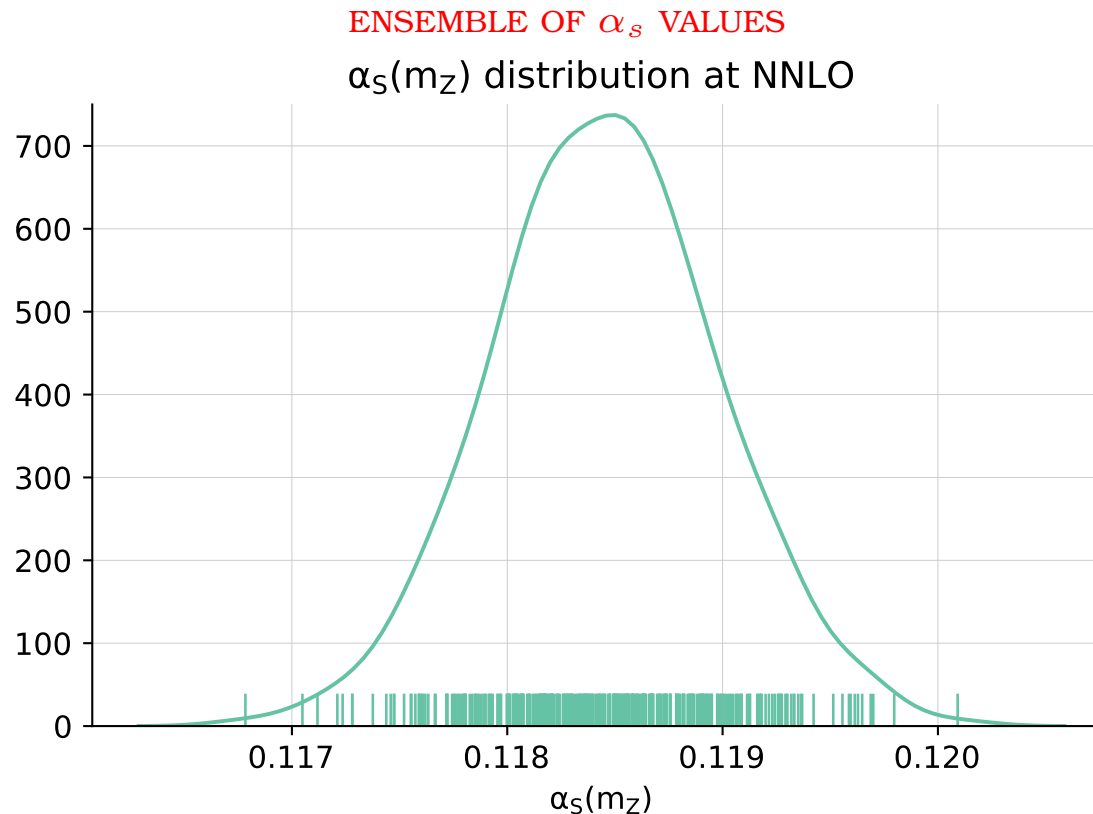
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DEPEND ON $\alpha_s \Rightarrow$ DIFFICULT TO TREAT AS CONTINUOUS PARAMETER
- SOLUTION DETERMINE BEST-FIT PDF REPLICA TO EACH DATA REPLICA FOR SEVERAL (DISCRETE) α_s VALUES: C-REPLICA
 - EACH C-REPLICA $\Rightarrow \chi^2$ PROFILE $\Rightarrow \alpha_s$ VALUE



THE CORRELATED REPLICA METHOD

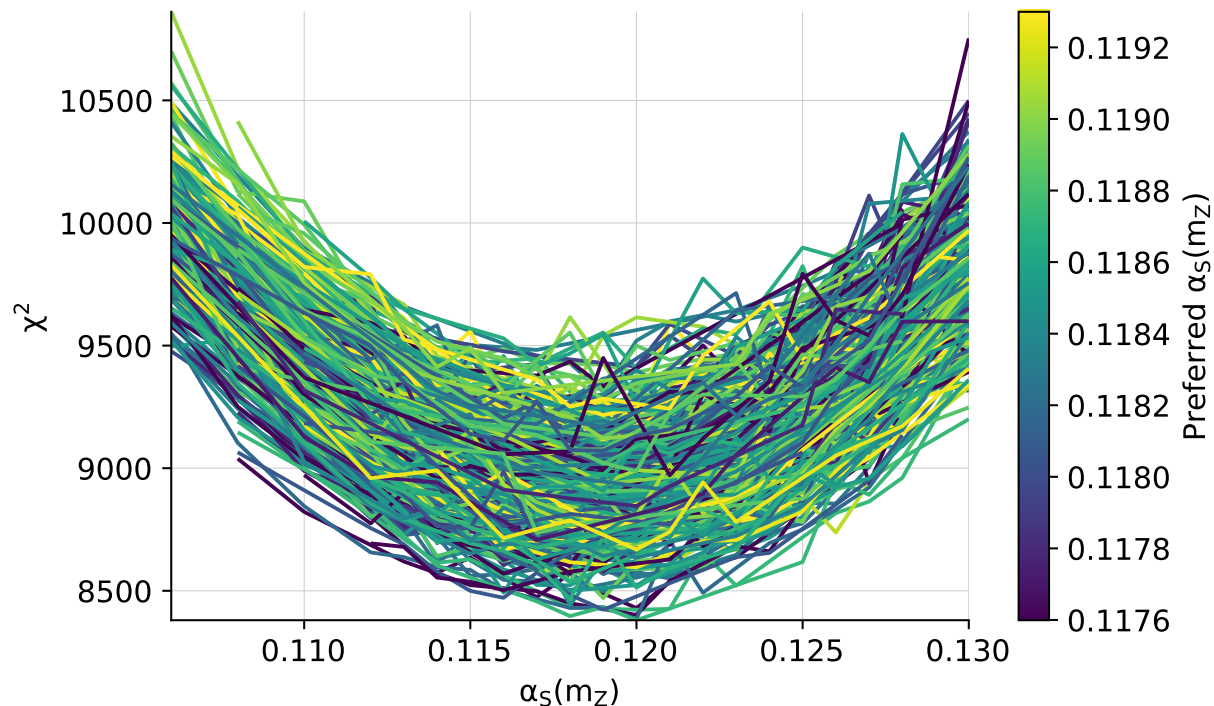
NNPDF3.1 (2018)

- NNPDF METHOD $\Rightarrow$ EACH PDF REPLICA FITTED BY GA TO DATA REPLICA
- IDEALLY PERFORM GENETIC MINIMIZATION IN (PDF, α_s) SPACE
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DEPEND ON $\alpha_s \Rightarrow$ DIFFICULT TO TREAT AS CONTINUOUS PARAMETER
- SOLUTION DETERMINE BEST-FIT PDF REPLICA TO EACH DATA REPLICA FOR SEVERAL (DISCRETE) α_s VALUES:
 - EACH C-REPLICA $\Rightarrow \chi^2$ PROFILE $\Rightarrow \alpha_s$ VALUE
 - EACH C-REPLICA $\Rightarrow$ BEST-FIT α_s REPLICA



α_s FROM CORRELATED REPLICAS

$\alpha_s(m_Z)$ distribution at NNLO



- NNPDF3.1 DATASET (ONLY NNLO JET DATA) $\Rightarrow$ 3979 DATAPOINTS
- 400 C-REPLICAS FOR 21 α_s VALUES: $\alpha_s(M_Z) = 0.106, 0.108, 0.102, 0.112, 0.113, 0.114, 0.115, 0.116, 0.117, 0.118, 0.119, 0.120, 0.121, 0.122, 0.123, 0.124, 0.125, 0.126, 0.127, 0.128, 0.130$
- EXPERIMENTAL UNCERTAINTY $\Leftrightarrow$ STANDARD DEV. OVER REPLICA SAMPLE

THE RESULT

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.11845 \pm 0.00052^{\text{exp}} (0.4\%)$$

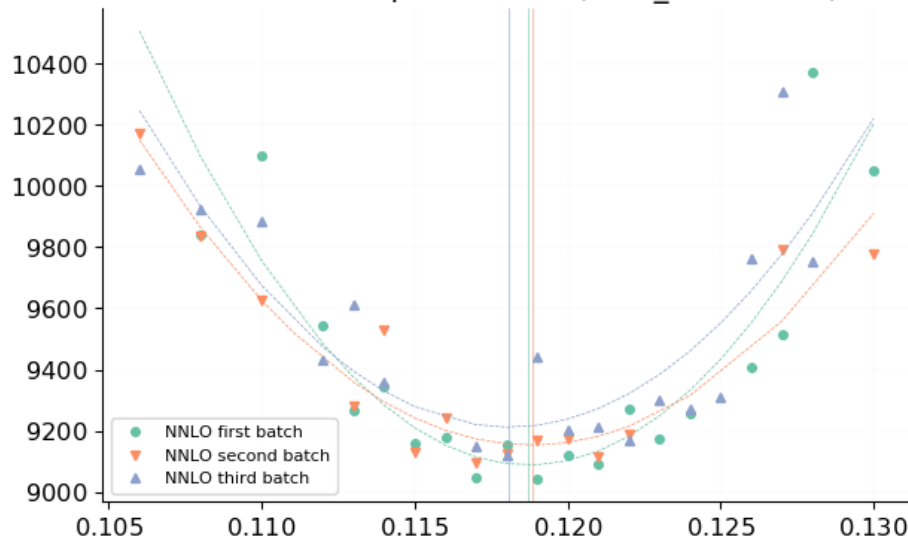
THE METHODOLOGY AND ITS UNCERTAINTIES

- WHY IS THE EXPERIMENTAL UNCERTAINTY SO SMALL?
- BECAUSE THERE IS A LOT OF INFORMATION IN C-REPLICAS

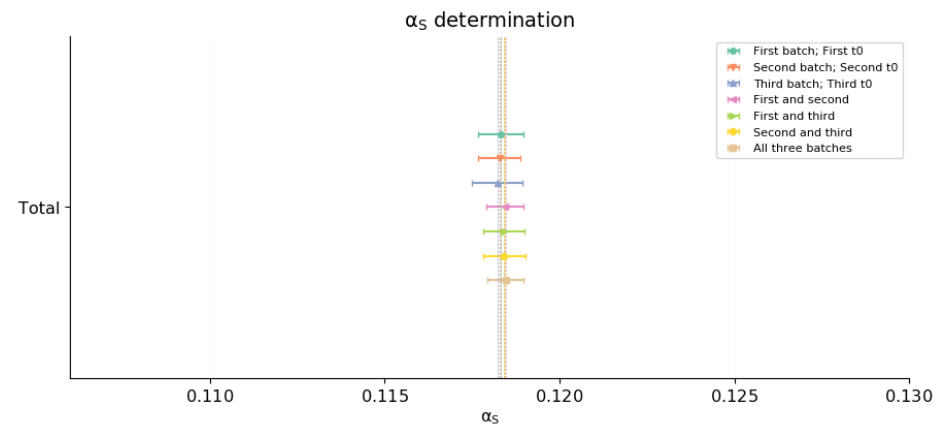
BATCH MINIMIZATION

DIFFERENT BATCHES FOR ONE REPLICA

Parabola example for Total (nnfit_index=102)



DEPENDENCE OF α_s ON BATCH CHOICE

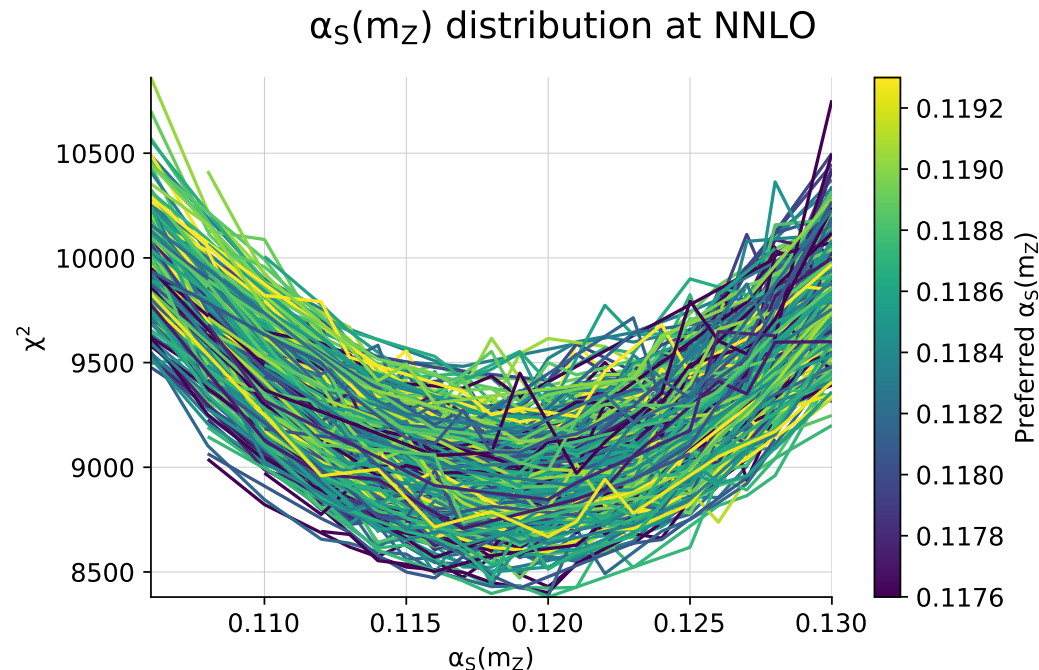


- EACH REPLICA FOR EACH α_s FITTED THREE TIMES (BATCHES)
- BEST FIT CHOSEN
- SINGLE, DOUBLE & TRIPLE BATCH COMPARED $\Rightarrow$
LITTLE DEPENDENCE OF CENTRAL VALUE, IMPROVEMENT OF UNCERTAINTIES
SINGLE BATCH $\sigma^{\text{exp}} = 0.0006 - 0.0007$; THREE BATCHES $\sigma^{\text{exp}} = 0.0005$

METHODOLOGICAL UNCERTAINTIES

FINITE-SIZE OF REPLICA SAMPLE

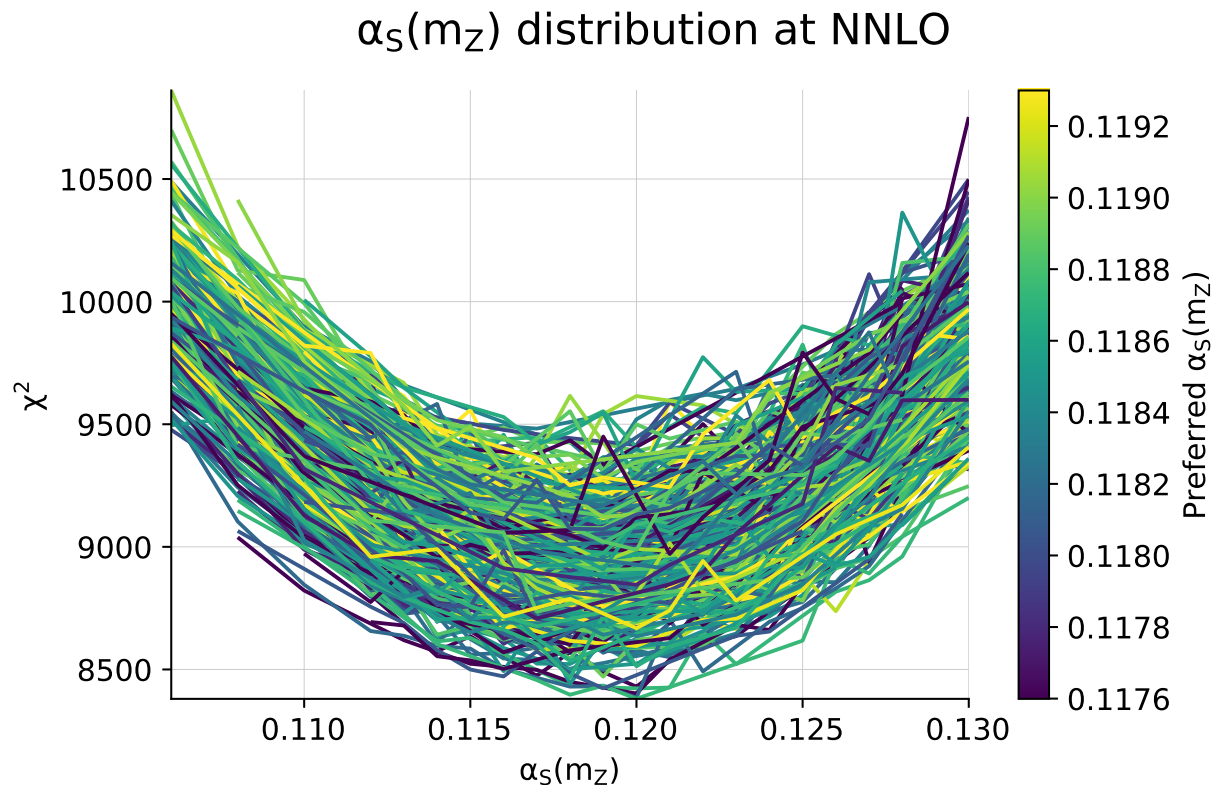
- ESTIMATED BY BOOTSTRAPPING: $\Delta^{\text{finite size}} = 0.00003$ (0.03%)
- DEPENDS ON REPLICA SELECTION $\Leftrightarrow$ ONLY INCLUDE A C-REPLICA IF FIT CONVERGED FOR AT LEAST $N_{\min}$ α_s VALUES
- MORE RESTRICTIVE SELECTION $\Rightarrow$ SMALLER σ^{exp} , LARGER $\Delta^{\text{finite size}} = 0.00009$
- WITH MOST RESTRICTIVE SELECTION ($N_{\min} = 18$, 12 SURVIVING REPLICAS)
 $\Delta^{\text{finite size}} = 0.00009$ (0.08%), BUT $\sigma^{\text{exp}} = 0.00031$ (0.3%)
- NO DEPENDENCE OF α_s VALUE ON CURVE SELECTION
- **DEFAULT: CONSERVATIVE** CHOICE $N_{\min} = 6$, 379 SURVIVING REPLICAS)
 $\Delta^{\text{finite size}} = 0.00003$ (0.03%), BUT $\sigma^{\text{exp}} = 0.00052$ (0.5%)
 MINIMIZES UNCERTAINTY ON UNCERTAINTY



METHODOLOGICAL UNCERTAINTIES

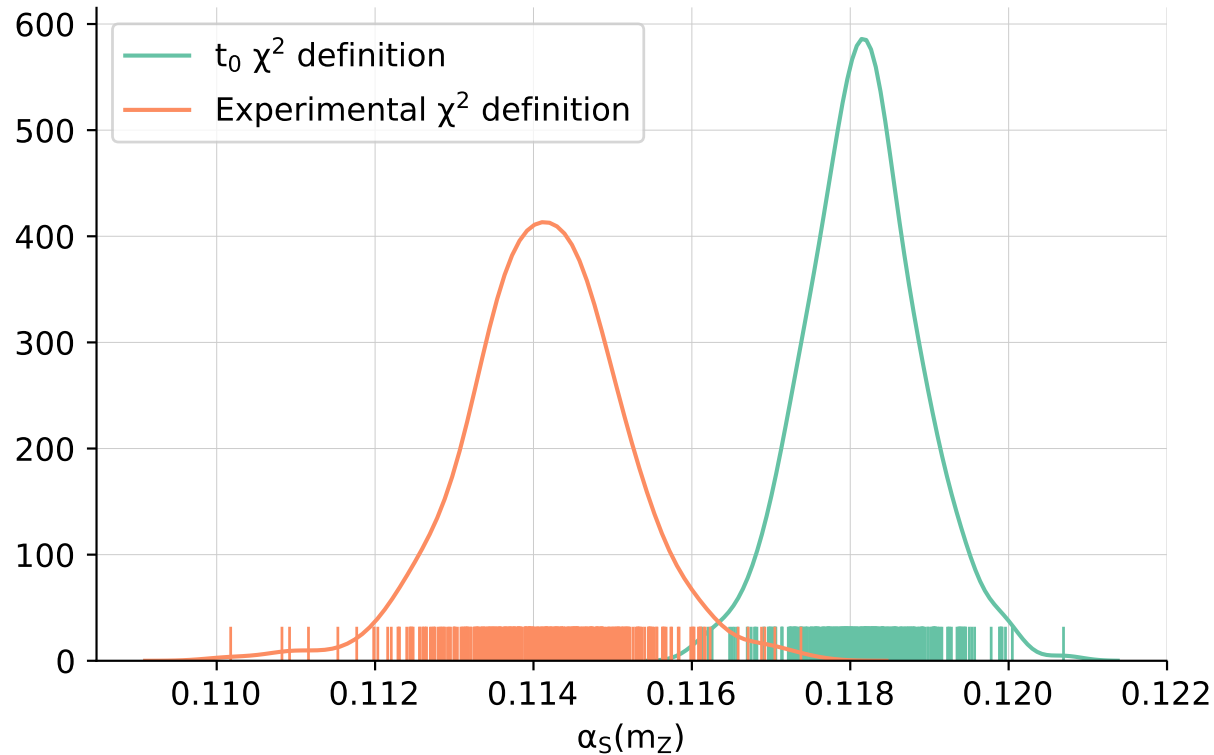
PARABOLIC FITTING

- FIT $\exp \alpha_s$ OR $\ln(\alpha_s + 1) \Rightarrow$ NO CHANGE
- REMOVE OUTER VALUES OF α_s , SYMMETRICALLY OR ASYMMETRICALLY $\Rightarrow$ LARGEST SHIFT $\Delta^{\text{parabolic}} = 0.00010$ (0.08%) $\Rightarrow$ DOMINANT METH. UNCERTAINTY
- NO EVIDENCE THAT CUBIC FIT IS BETTER THAN QUADRATIC



METHODOLOGICAL UNCERTAINTIES

NORMALIZATION UNCERTAINTIES



- **MULTIPLICATIVE** UNCERTAINTIES (ALL HADRON COLLIDER SYST.)
⇒ t_0 METHOD, **REQUIRES PRIOR** FIT
- **GROSSLY BIASED** IF **EXPERIMENTAL COVARIANCE** MATRIX USED
- PRIOR FIT TAKEN FROM **THREE DIFFERENT BATCHES**
⇒ **VERY SMALL SHIFT** $\Delta^{t_0} = 0.00004$ (0.03%)

TOTAL METHODOLOGICAL UNCERTAINTY
 $\sigma^{\text{meth}} = 0.00011$ (0.09%)

MISSING HIGHER ORDER UNCERTAINTIES

$$\alpha_s^{\text{NLO}}(M_Z) = 0.11845 \pm 0.00052^{\text{exp}} \quad (0.4\%)$$

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.12067 \pm 0.00065^{\text{exp}} \quad (0.5\%)$$

$$\Delta\alpha_s^{\text{NLO-NNLO}} = 0.0022$$

- **CACCIARI-HOUDEAU:** $\alpha_s^{\text{N}^k\text{LO}} = \sum_{n=0}^k c_n [\alpha_s^{\text{true}}]^n$;
BAYESIAN ESTIMATE FOR RANGE OF NEXT MISSING COEFFICIENTS
- **DIFFICULTIES:**
 - α_s^{true} NOT KNOWN $\Rightarrow$ VARY IN WIDE RANGE
 - α_s^{LO} EFFECTIVELY NOT KNOWN $\Rightarrow$ VARY IN WIDE RANGE
 - IS IT **MEANINGFUL FOR A COMBINATION** OF PROCESSES?

$$\Delta\alpha_s^{\text{NLO, CH}} = 0.003$$

$$\Delta\alpha_s^{\text{NNLO, CH}} = 0.0004$$

NOTE **AT NLO AGREES WELL** WITH KNOWN NNLO-NLO SHIFT!

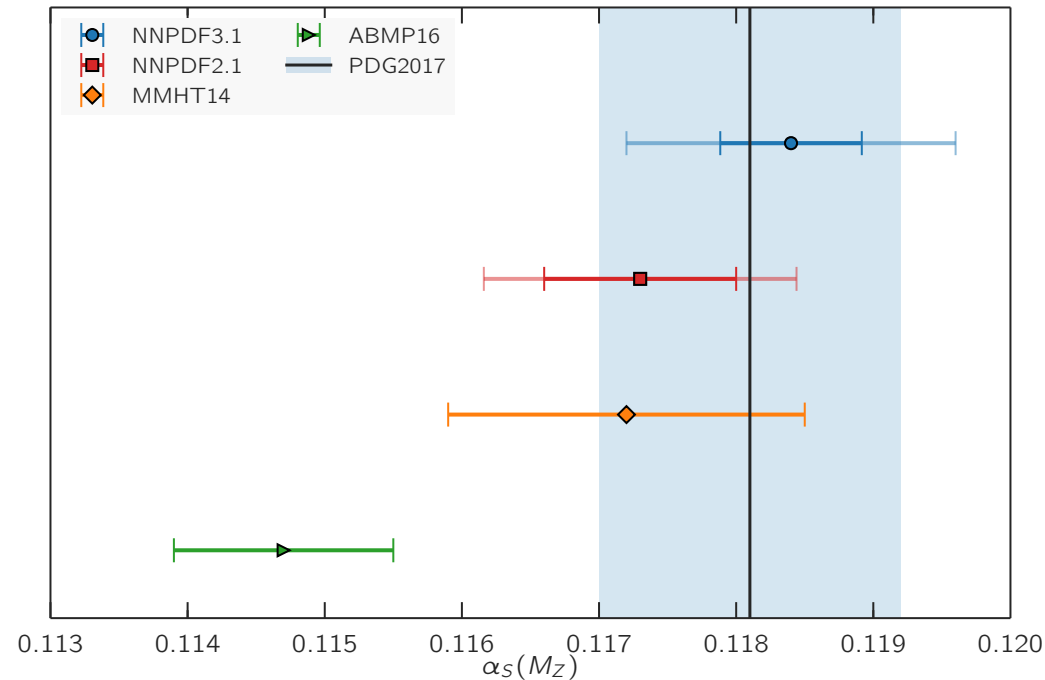
TOTAL THEORY UNCERTAINTY
OUR VERY CONSERVATIVE ESTIMATE:

$$\sigma^{\text{th}} = 0.0011 \quad (0.9\%)$$

- **HALF THE NLO-NNLO** SHIFT
- **THREE TIMES THE CH** ESTIMATE

FINAL RESULT & COMPARISON

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.1185 \pm 0.0005^{\text{exp}} \pm 0.0001^{\text{meth}} \pm 0.0011^{\text{th}} = 0.1185 \pm 0.0012 \text{ (1\%)}$$

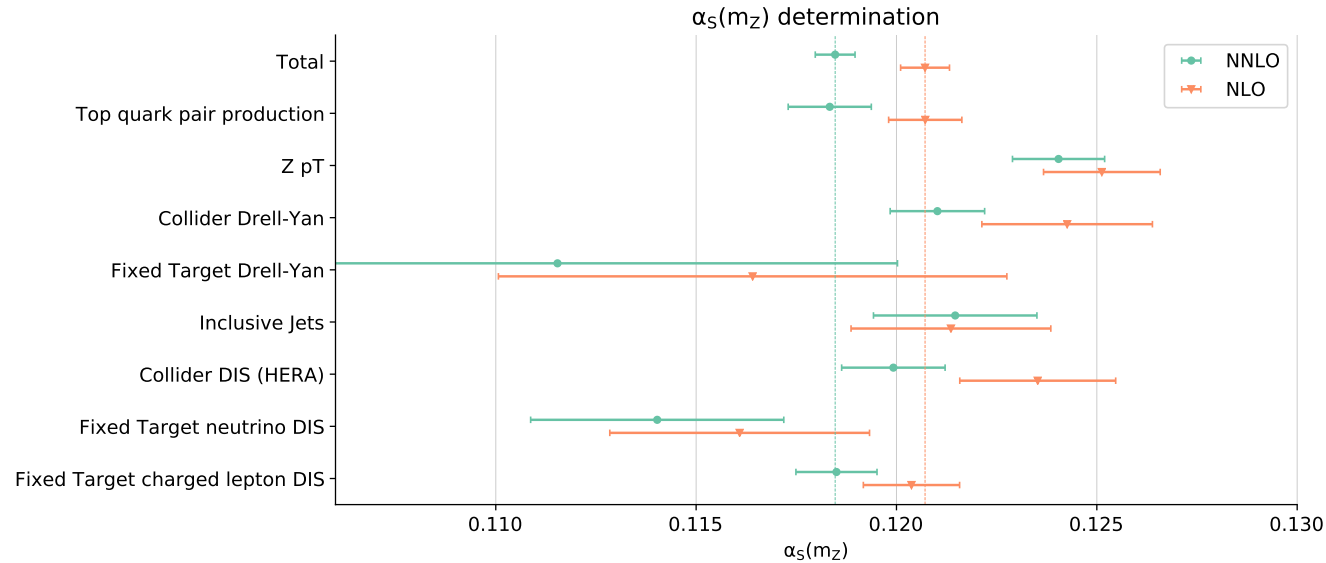


- **SIGNIFICANTLY SMALLER EXP. UNCERTAINTY** IN COMPARISON TO PREVIOUS NNPDF2.1 DETERMINATION (DESPITE MORE CONSERVATIVE ESTIMATE)
- SOMEWHAT **LARGER CENTRAL VALUE** THAN MMHT

THE IMPACT OF LHC DATA

PULLS FROM DATA SUBSETS

α_s values from minimization of partial χ^2 for each c-replica



LESSONS

- NO SIGNS OF TENSION OR INCOMPATIBILITY
- HADRON COLLIDER DATA PULL TOWARDS LARGER α_s

CAVEATS

- PARTIAL VALUES ARE NOT PARTIAL BEST-FITS (PDFs COME FROM GLOBAL FIT)
- REPLICAS SELECTION BASED ON GLOBAL SET (SOME PARABOLAE $\Rightarrow$ NO MINIMUM)
- MISSING SYSTEMATIC CORRELATION BETWEEN DATASETS
- PULLS CORRELATED THROUGH UNDERLYING PDFs

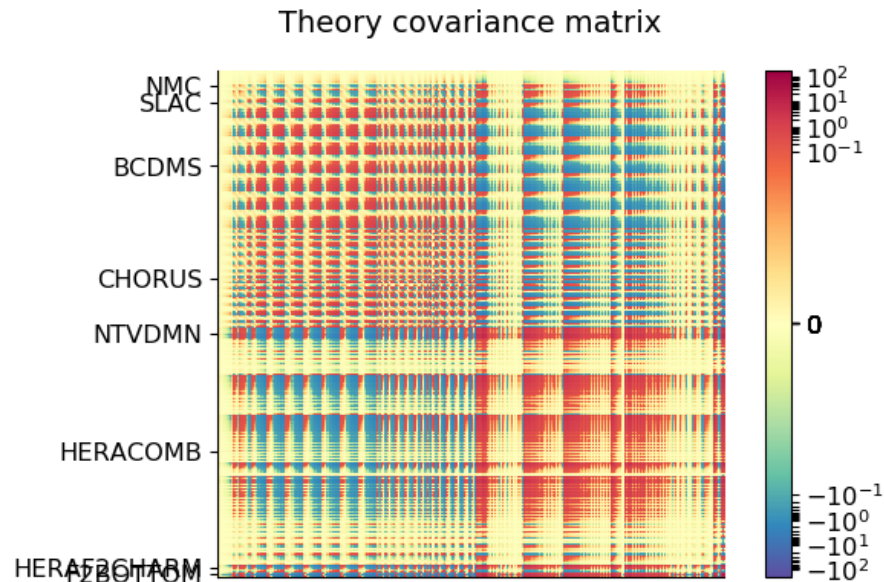
LESSONS LEARNT

- LHC DATA $\Rightarrow$ PDF + α_s UNCERTAINTIES AT THE SUBPERCENT LEVEL
- LHC DATA $\Rightarrow$ LARGER α_s VALUE
- PDF-SENSITIVE α_s DETERMINATION WITHOUT SIMULTANEOUS PDF FIT IS DANGEROUS AND LESS ACCURATE
- MORE GLOBAL DATASET $\Rightarrow$ GREATER PERTURBATIVE STABILITY

CAN WE DO BETTER?

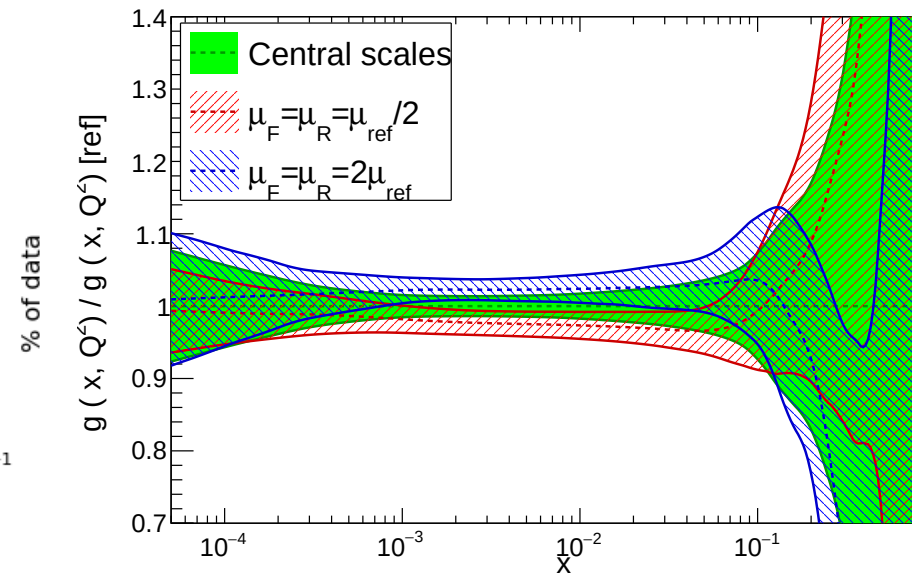
THEORETICAL UNCERTAINTIES: MHO

THEORY UNCERTAINTIES: COVARIANCE MATRIX



GLUON PDF: SCALE VARIATION

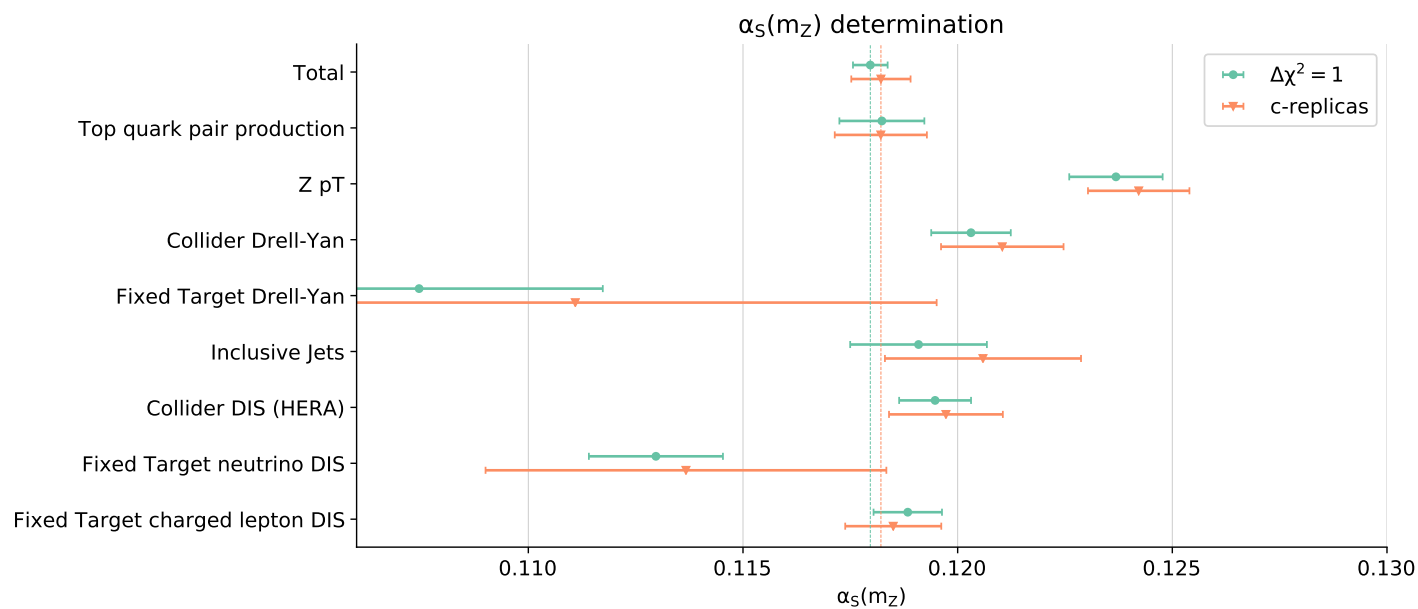
NNPDF3.1 DIS-only NNLO, $Q = 100$ GeV



- PDF FITS WITH SCALE VARIED THEORY
- THEORY UNCERTAINTIES (FROM SCALE OR OTHERWISE)
IN COVARIANCE MATRIX
GENERALIZABLE TO OTHER THEORY UNCERTAINTIES
- THEORY UNCERTAINTIES INCLUDED IN PDF UNCERTAINTIES

EXTRAS

COMPARISON WITH THE OLD METHOD



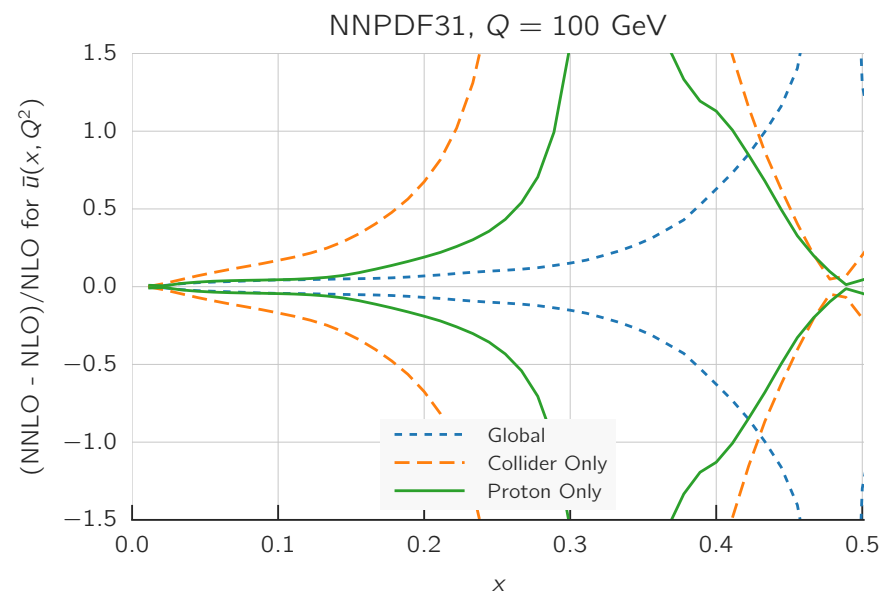
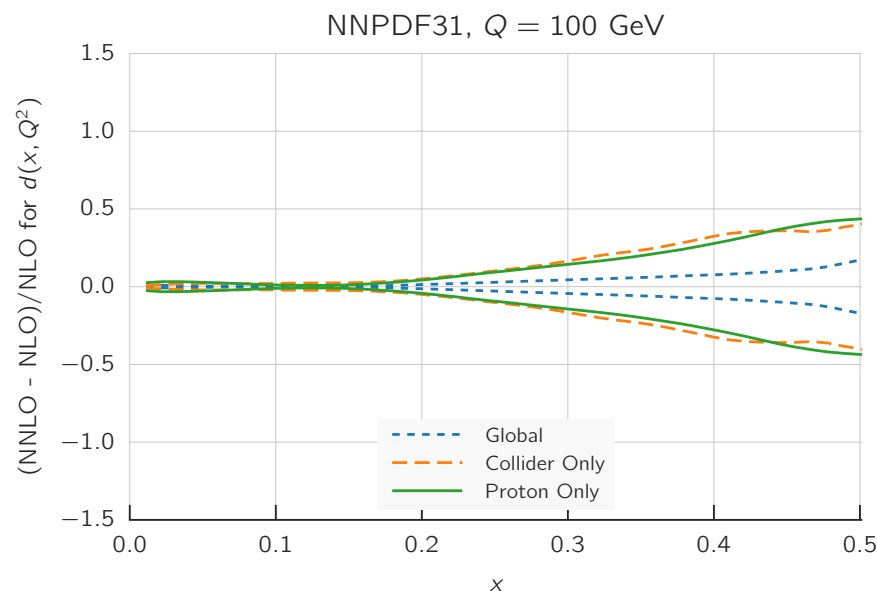
- GOOD AGREEMENT OF CENTRAL VALUES
- OLD METHOD $\Rightarrow$ SOMEWHAT UNDERESTIMATED UNCERTAINTIES

PERTURBATIVE STABILITY

GLOBAL VS RESTRICTED DATASETS

DOWN

ANTIUP



- NLO-NNLO **SHIFTS SMALLER** WITH LARGER DATASET
- **GREATER STABILITY** OF α_s ALSO OBSERVED