DIFFERENTIAL SOFT RESUMMATION

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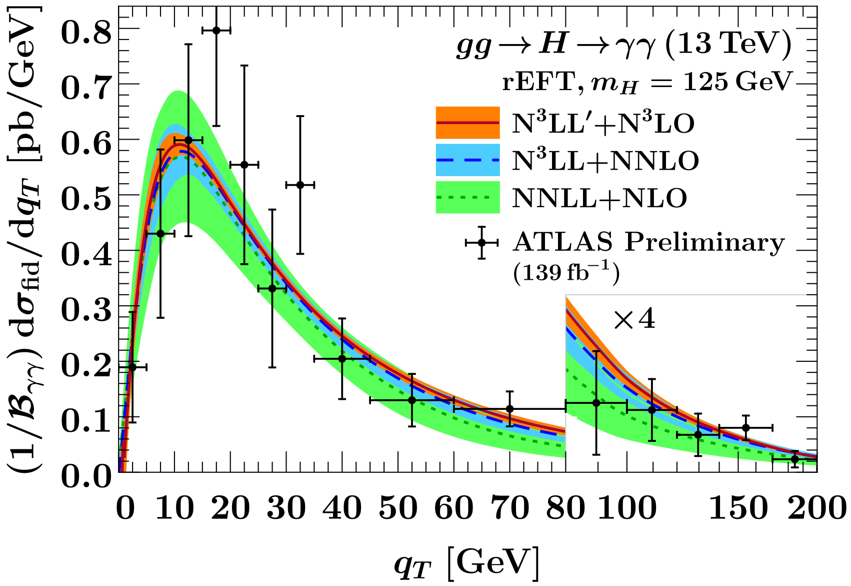
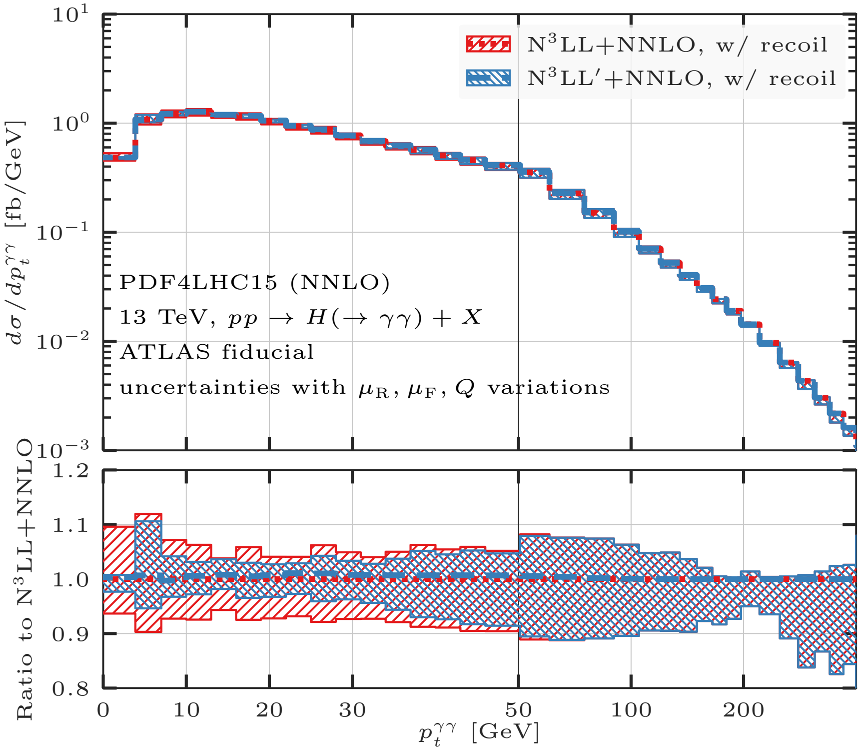
RESUMMATION TODAY

- FOCUS ON HIGGS SIGNAL+ BACKGROUND
- RESUMMATION OF FIDUCIAL OBSERVABLES
- HIGH LOGARITHMIC ACCURACY
- MATCHING OF FIXED ORDER TO TRANSVERSE MOMENTUM RESUMMATION

HIGGS IN GLUON FUSION



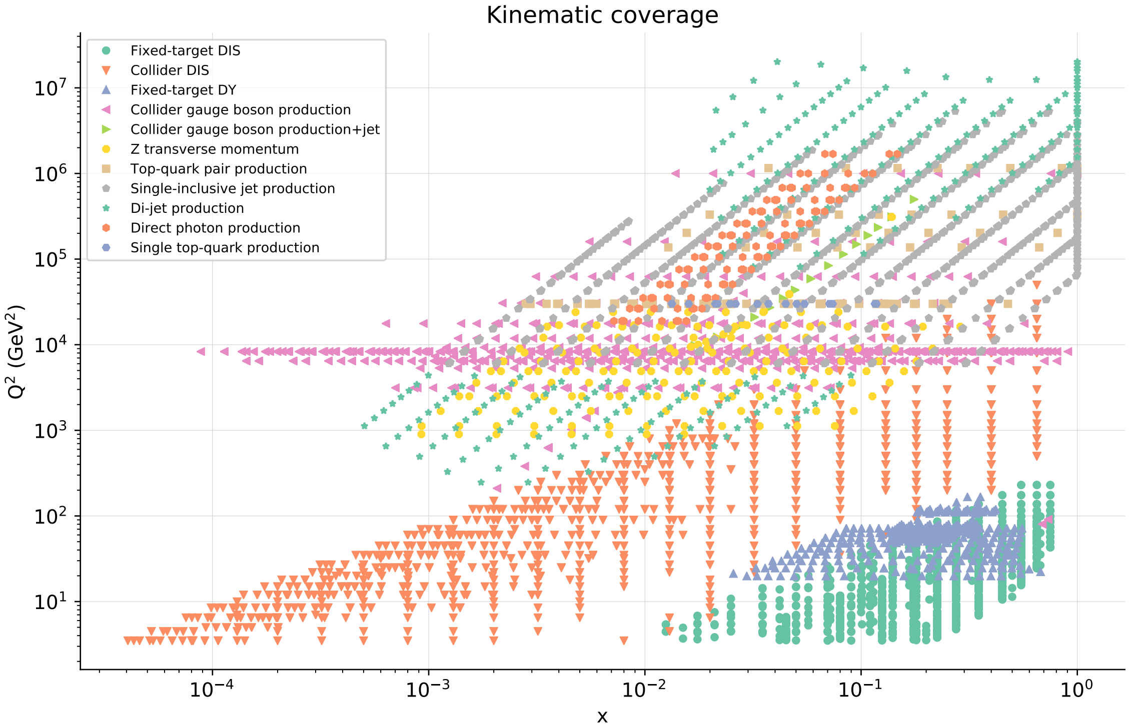
(Billie et al., 2021)



(Re, Rottoli, Torrielli, 2021)

PRECISION SM PHYSICS
PDF DETERMINATION

THE NNPDF4.0 DATASET



(Ball et al, 2021)

- W AND Z RAPIDITY DISTRIBUTIONS PLAY A DOMINANT ROLE
- Z p_T ALSO RELEVANT
- VERY HIGH EXPERIMENTAL PRECISION
- TREATED WITH PURE FIXED ORDER NNLO QCD
- WHAT ABOUT SOFT RESUMMATION?

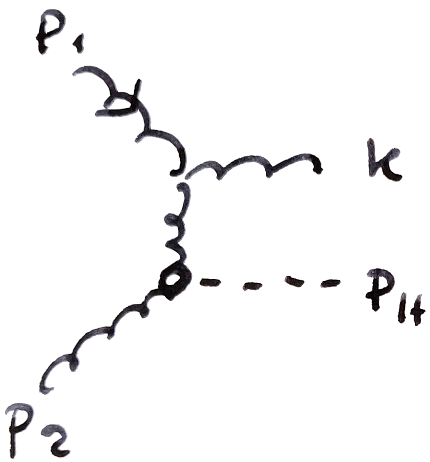
SOFT RESUMMATION AT THE DIFFERENTIAL LEVEL
WHAT DO WE KNOW?
DOUBLE- AND TRIPLE-DIFFERENTIAL DRELL-YAN/HIGGS

- TRANSVERSE MOMENTUM DISTRIBUTIONS
  - RESUM $\ln N$ (MELLIN) AT FIXED p_T
  - NLL ACCURACY (De Florian, Kulesza, Vogelsang, 2006)
  - GENERAL (SF, Ridolfi, Rota, 2021)
- b RAPIDITY DISTRIBUTIONS: DOUBLE MELLIN $N_1 = N + ib$, $N_2 = N - ib$
  - RESUM $\ln N$ AT FIXED b: b DEPENDENCE SUBLEADING (Bolzoni, 2006; Becher, Neubert, 2008)
  - RESUM $\ln N_1$, $\ln N_2$ IN DOUBLE SOFT LIMIT N_1, $N_2 \to \infty$: FOLLOWS FROM INCLUSIVE (Catani, Trentadue, 1989; Westmark, Owens, 2017; Ravindran et al, 2018)
  - RESUM $\ln N_1$ IN SINGLE SOFT $N_1 \to \infty$, N_2 FIXED: SCET (MOMENTUM SPACE) (Lustermans, Michel, Tackmann, 2019, unpublished)
- FULLY DIFFERENTIAL
  - UNKNOWN

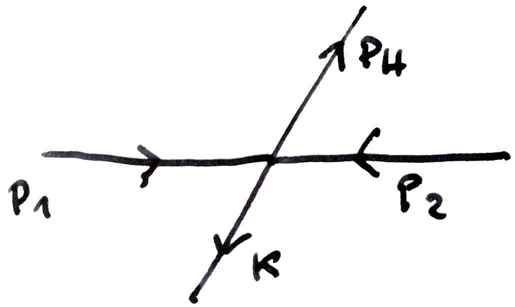
$2 \to 2$ KINEMATICS

COLORLESS FINAL STATE: (SAY) HIGGS IN GLUON FUSION

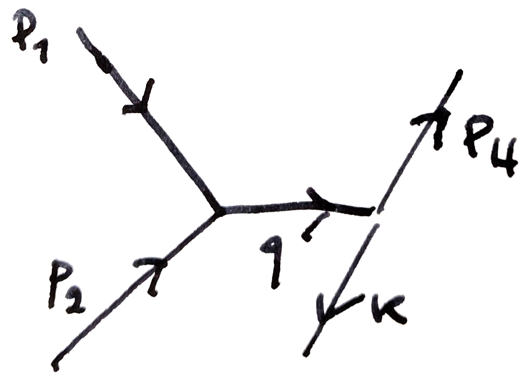
FEYNMAN DIAGRAM



KINEMATICS



KINEMATIC DIAGRAM



MOMENTA

$$p_1 + p_2 = p_h + k$$

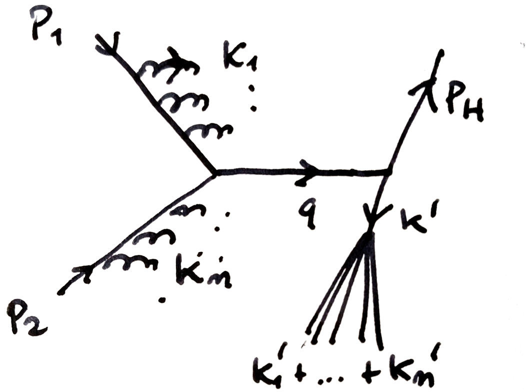
PHASE SPACE

$$d\phi_2(p_1 + p_2; p_H, k) = \int \frac{dq^2}{2\pi} d\phi_1(p_1 + p_2; q) d\phi_2(q; p_H, k)$$

$2 \to 2 + X$ KINEMATICS:

SOFT LIMITS

KINEMATIC DIAGRAM



$$d\phi_{n+m+2}(p_1+p_2;p_H,k_1,\ldots,k_n,k'_1,\ldots,k'_{m+1}) =$$
$$\int \frac{dq^2}{2\pi} d\phi_{n+1}(p_1+p_2;q,k_1,\ldots,k_n) \int \frac{dk'^2}{2\pi}\, d\phi_2(q;p_H,k')\, d\phi_{m+1}(k';k'_1,\ldots,k'_{m+1})$$

- MOMENTA $k_1,\ldots k_n$ ARE SOFT
- MOMENTA $k'_1,\ldots k'_{m+1}$ ARE COLLINEAR (m COLLINEAR EMISSIONS), RECOILING AGAINST "HIGGS"
- PHASE SPACE SPLIT ITERATIVELY (SF, Ridolfi, 2003; Jing, Shen, Guo 2021)
  - $d\phi_{n+1}$: TWO INCOMING INTO ONE OFF-SHELL q & n SOFT
  - TWO-BODY INCOMING q INTO HIGGS + OFF-SHELL k'
  - OFF-SHELL k' INTO $m+1$ COLLINEAR

RG APPROACH TO SOFT RESUMMATION

THE INCLUSIVE CASE (SF, Ridolfi, 2003)

RESUMMATION OF THE INCLUSIVE PARTONIC CROSS SECTION $\hat{\sigma}(Q^2, x)$

HIGGS: $Q^2 = m_H^2$, $x = \frac{m_H^2}{\hat{s}}$

1. KINEMATICS: SOFT LIMIT $\to$ PARTONIC CROSS SECTION ONLY DEPENDS ON SCALE VARIABLE $Q^2(1-x)^2$(HIGGS, DY; $Q^2(1-x)$ FOR DIS)
2. RG: RG INVARIANT QUANTITY CAN ONLY DEPEND ON SCALE THROUGH α_s

- USE MELLIN SPACE $\Rightarrow \hat{\sigma} = \hat{\sigma}(Q^2, N)$
- SOFT LIMIT $\Rightarrow \hat{\sigma}(Q^2, N) = H\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) J\left(\frac{Q^2/N^2}{\mu^2}, \alpha_s(\mu^2)\right)$
- RG INVARIANT: $\gamma^{\rm phys} = \frac{d}{d\ln Q^2}\ln\hat{\sigma} = \gamma^{\rm c} + \gamma^{\rm l}$; $\gamma^{\rm c} = \frac{d}{d\ln Q^2}\ln H$ $\gamma^{\rm l} = \frac{d}{d\ln Q^2}\ln J$
- RGE $\frac{d}{d\mu^2}\gamma^{\rm phys} = 0 \Leftrightarrow \frac{d}{d\mu^2}\gamma^{\rm l} = -\frac{d}{d\mu^2}\gamma^{\rm c} \equiv g(\alpha_s(\mu^2))$

RGE SOLUTION: THE RESUMMED CROSS-SECTION

$$\hat{\sigma}\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s(Q^2)\right) = \sigma^0\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s(Q^2)\right) C_{\rm res}(N, \frac{Q^2}{\mu_F^2}, \alpha_s(Q^2))$$

$$= \sigma^0\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s(Q^2)\right) H(\alpha_s(Q^2)) \exp\int_{\mu_F^2}^{Q^2}\frac{d\mu^2}{\mu^2}\int_1^{N^2}\frac{dn}{n} g\left[\alpha_s(\mu^2/n)\right]$$

THE SOFT-RESUMMED CROSS SECTION (INCLUSIVE)

$$C_{\rm res}(N,\alpha_s(Q^2)) = g_0(\alpha_s(Q^2))\exp\left[\int_1^{N^2}\frac{dn}{n}\int_{Q^2 n}^{Q^2}\frac{d\mu^2}{\mu^2}\, g\left[\alpha_s(\mu^2/n)\right]\right]$$

$$= \hat{g}_0(\alpha_s)\exp\left[2\int_0^1 dx\,\frac{x^{N-1}-1}{1-x}\int_{Q^2}^{Q^2(1-x)^2}\frac{d\mu^2}{\mu^2}\,\hat{g}\left[\alpha_s(\mu^2)\right]\right]$$

$$= \hat{g}_0(\alpha_s)\exp\left[2\int_0^1 dx\,\frac{x^{N-1}-1}{1-x}\int_{Q^2}^{(1-z)^2 Q^2}\frac{dq^2}{q^2}A\left(\alpha_s\left(q^2\right)\right)+B\left(\alpha_s\left((1-z)^2 Q^2\right)\right)\right]$$

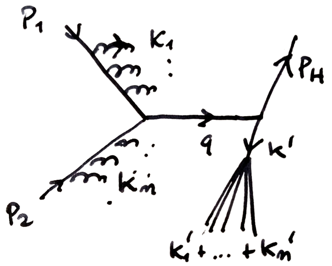
- PDFs EVALUATED AT $\mu_F^2 = Q^2$
- g DETERMINED BY MATCHING TO FIXED ORDER

SWOT RG

- STRENGTH: SIMPLE INTERPRETATION AND DERIVATION
- WEAKNESS: RESUMMATION COEFFICIENTS DETERMINED BY MATCHING TO FIXED ORDER
- OPPORTUNITIES: GENERALIZATION TO MULTISCALE

TRANSVERSE MOMENTUM DISTRIBUTIONS

(SF, Ridolfi, Rota, 2021)



$$d\phi_{n+m+2}(p_1+p_2; p_H, k_1, \dots, k_n, k'_1, \dots, k'_{m+1})$$
$$= \int \frac{dq^2}{2\pi} d\phi_{n+1}(p_1+p_2; q, k_1, \dots, k_n)$$
$$\int \frac{dk'^2}{2\pi} d\phi_2(q; p_H, k')\, d\phi_{m+1}(k'; k'_1, \dots, k'_{m+1})$$

SOFT LIMIT

FIXED p_T, $\hat{s} \to \hat{s}^{\min} = \left(\sqrt{m_H^2 + p_T^2} + \sqrt{p_T^2}\right)^2 \equiv Q^2$; $x \equiv \frac{Q^2}{\hat{s}}$

FIXED (Q^2, Qp_T), ($x \to 1 \leftrightarrow N \to \infty$ (MELLIN))

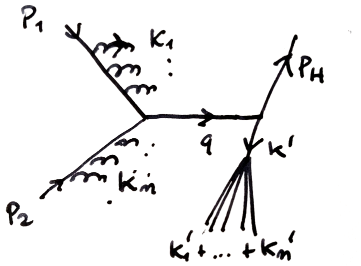
SOFT KINEMATICS

q^2 SQUEEZED TO ITS MINIMUM $q^2 \to {q^2}_{\min} = Q^2$; k'^2 FORCED ON-SHELL $k'^2 \to 0$
$\Rightarrow$ HIGGS $p_z \to 0$

SOFT PHASE SPACE

- $Q^2 \le q^2 \le \hat{s} \Leftrightarrow q^2 - Q^2 = Q^2\left(u\frac{1-x}{x}\right)$, $0 \le u \le 1$
- $0 \le k'^2 \le Qp_T(1-x)u \Leftrightarrow k'^2 = uvQp_T(1-x)$, $0 \le v \le 1$
- $dq^2 dk'^2 \to du dv$

TRANSVERSE MOMENTUM DISTRIBUTIONS



$$d\phi_{n+m+2}(p_1+p_2;p_H,k_1,\ldots,k_n,k'_1,\ldots,k'_{m+1})$$
$$\sim \int dudvd\phi_{n+1}(p_1+p_2;q,k_1,\ldots,k_n)$$
$$d\phi_{m+1}(k';k'_1,\ldots,k'_{m+1})$$

SOFT PHASE SPACES AND SCALES

- $d\phi_{n+1}(p_1+p_2;q,k_1,\ldots,k_n)$ INCLUSIVE HIGGS-LIKE, SCALE $\frac{(s-q^2)^2}{q^2} = Q^2(1-x)^2$
- $d\phi_{m+1}(k';k'_1,\ldots,k'_{m+1})$ DIS-LIKE, SCALE ${k'}^2 = Qp_T(1-x)$

SOFT RESUMMATION

$$C(N,Q^2/\mu^2,Qp_T/\mu^2,\alpha_s(\mu^2)) = C^{(c)}\left(\alpha_s(Q^2),\frac{Q^2}{\mu^2}\right)$$

$$\times \exp\left[\int_1^{N^2}\frac{dn_1}{n_1}\int_{n_1\mu^2}^{Q^2}\frac{dk_1^2}{k_1^2}\bar{g}_1^{(i)}(\alpha_s(k_1^2)) + \int_1^{N}\frac{dn_2}{n_2}\int_{n_2\mu^2}^{Qp_T}\frac{dk_2^2}{k_2^2}\bar{g}_2^{(j)}(\alpha_s(k_2^2))\right]$$

TRANSVERSE MOMENTUM DISTRIBUTIONS

SOFT RESUMMATION

$$C(N, Q^2/\mu^2, Qp_T/\mu^2, \alpha_s(\mu^2)) = g_0\left(\alpha_s(Q^2)\right)$$
$$\times \exp\left\{ \int_0^1 dx \, \frac{x^{N-1}-1}{1-x} \left[D[\alpha_s(Q^2(1-x)^2)] + \int_{\mu^2}^{Q^2(1-x)^2} \frac{dk^2}{k^2} A[\alpha_s(k^2]\right] \right.$$
$$\left. + \int_0^1 dx \, \frac{x^{N-1}-1}{1-x} \left[B[Qp_T(1-x)] + \int_{\mu^2}^{Qp_T(1-x)} \frac{dk^2}{k^2} A[\alpha_s(k^2)]\right] \right\}$$

NLL RESULT

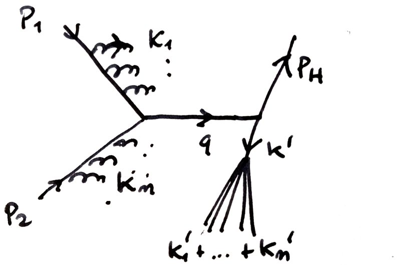
(De Florian, Kulesza, Vogelsang, 2006)

$$C(N, Q^2/\mu^2, Qp_T/\mu^2, \alpha_s(\mu^2)) = g_0(\alpha_s(Q^2))$$
$$\times \exp\left\{ 2 \int_0^1 dx \, \frac{x^{N-1}-1}{1-x} \int_{\mu^2}^{Qp_T(1-x)^2} \frac{dq^2}{q^2} A(\alpha_s(q^2)) \right.$$
$$\left. + \int_0^1 dx \, \frac{x^{N-1}-1}{1-x} \left[\int_{Qp_T(1-x)^2}^{Qp_T(1-x)} \frac{dq^2}{q^2} A(\alpha_s(q^2)) + B(\alpha_s(Qp_T(1-x)))\right] + \int_0^1 dx \, \frac{x^{N-1}-1}{1-x} A \ln \frac{p_T}{Q} \right\}$$

- $\ln p_t$ TERM EFFECTS SCALE CHANGE TO NLL ACCURACY
- AGREEMENT AT NLL (NOTE $D = 0$ AT NLL)
- NEW RESULT GENERALIZES TO ALL LOG ORDERS

RAPIDITY DISTRIBUTIONS

(De Ros, SF, Tagliabue, WIP)



$$d\phi_{n+m+2}(p_1+p_2;p_H,k_1,\dots,k_n,k'_1,\dots,k'_{m+1})$$
$$=\int \frac{dq^2}{2\pi} d\phi_{n+1}(p_1+p_2;q,k_1,\dots,k_n)$$
$$\int \frac{dk'^2}{2\pi}\, d\phi_2(q;p_H,k')\, d\phi_{m+1}(k';k'_1,\dots,k'_{m+1})$$

(PARTON) KINEMATICS

$$p_H = \left(\sqrt{m_H^2+p_T^2}\cosh y, \vec{p}_T, \sqrt{m_H^2+p_T^2}\sinh y\right),\ x_1=\sqrt{x}e^{y}\ x_2=\sqrt{x}e^{-y} \Leftrightarrow x = x_1 x_2,$$
$$(Q^2=m_H^2,\ x\equiv \frac{Q^2}{\hat{s}}),\ e^{2y}=\frac{x_1}{x_2} \Leftrightarrow \sqrt{x}\le e^{y}\le \frac{1}{\sqrt{x}}$$

SOFT LIMITS

- SINGLE SOFT: FIXED y, $\hat{s}\to \hat{s}^{\min}=\left(\sqrt{m_H^2+p_z^2}+\sqrt{p_z^2}\right)^2$
  $\Rightarrow$ FIXED (Q^2, x_2), $x_1\to 1 \leftrightarrow$ FIXED N_2, $N_1\to\infty$ (MELLIN)
- DOUBLE SOFT: $y\to y^{\min}=0$, $\hat{s}\to\hat{s}^{\min}=m_H^2$
  $\Rightarrow$ FIXED Q^2, $x_1\to 1$, $x_2\to 1 \leftrightarrow N_1\to\infty$, $N_2\to\infty$ (MELLIN)

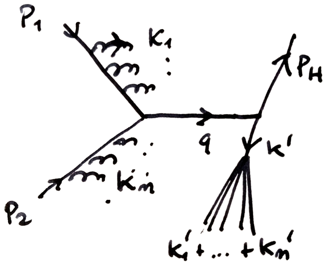
SOFT KINEMATICS

- SINGLE SOFT: q^2 SQUEEZED TO ITS MINIMUM $q^2\to {q^2}_{\min}$; k'^2 FORCED ON-SHELL $k'^2\to 0$
  $\Rightarrow$ HIGGS $p_T\to 0$
- DOUBLE SOFT: q^2 SQUEEZED TO ITS ABSOLUTE MINIMUM $q^2\to m_H^2$; k'^2 FORCED ON-SHELL $k'^2\to 0$
  $\Rightarrow$ HIGGS $p_T\to 0$ $p_z\to 0$

SOFT PHASE SPACE

- $x_1^2\hat{s}\le q^2\le\hat{s} \Leftrightarrow q^2-\hat{s}x_1^2=\hat{s}u(1-x_1^2)$, $0\le u\le 1$
- $0\le k'^2\le \hat{s}(1-x_1)(1-x_2) \Leftrightarrow k'^2=uv\hat{s}(1-x_1)(1-x_2)$, $0\le v\le 1$
- $dq^2dk'^2\to du dv$

RAPIDITY DISTRIBUTIONS



$$d\phi_{n+m+2}(p_1+p_2;p_H,k_1,\ldots,k_n,k'_1,\ldots,k'_{m+1})$$
$$\sim \int du dv d\phi_{n+1}(p_1+p_2;q,k_1,\ldots,k_n)$$
$$d\phi_{m+1}(k';k'_1,\ldots,k'_{m+1})$$

SOFT PHASE SPACES AND SCALES

- $d\phi_{n+1}(p_1+p_2;q,k_1,\ldots,k_n)$ INCLUSIVE HIGGS-LIKE; SCALE $\frac{(s-q^2)^2}{q^2}=Q^2(1-x_1)^2$
- $d\phi_{m+1}(k';k'_1,\ldots,k'_{m+1})$ DIS-LIKE; SCALE ${k'}^2=Q^2(1-x_1)(1-x_2)\to Q^2(1-x_1)$

SOFT RESUMMATION

RESUMMED EXPONENT

$$\ln C(N_1,N_2,Q^2/\mu^2,\alpha_s(\mu^2))=\left[\int_1^{N_1^2}\frac{dn}{n}\int_{n\mu^2}^{Q^2}\frac{dk_1^2}{k_1^2}\bar{g}_1^{(i)}(\alpha_s(k_1^2))+\int_1^{N_1N_2}\frac{dn'}{n'}\int_{n'\mu^2}^{Q^2}\frac{dk_2^2}{k_2^2}\bar{g}_2^{(j)}(\alpha_s(k_2^2))\right]$$

+non log.

RAPIDITY DISTRIBUTIONS

DOUBLE SOFT RESUMMATION

HIGGS-LIKE TERM SUBLEADING; DIS-LIKE SURVIVES:

$$C(N_1, N_2, Q^2/\mu^2, \alpha_s(\mu^2)) = g_0\left(\alpha_s(Q^2)\right)$$
$$\times \exp \int_0^1 dx_1 dx_2 \,\frac{x_1^{N_1-1}-1}{1-x_1}\,\frac{x_2^{N_2-1}-1}{1-x_2}\left[D[\alpha_s(Q^2(1-x_1)(1-x_2))] + \int_{\mu^2}^{Q^2(1-x_1)(1-x_2)} \frac{dk^2}{k^2} A[\alpha_s(k^2]\right]$$

- DIFFERENTIAL OBTAINED FROM INCLUSIVE BY $N^2 \to N_1 N_2$: $\ln \frac{Q^2}{N_1 N_2}$ RESUMMATION
- AGREEMENT WITH Catani, Trentadue, 1989; Westmark, Owens, 2017; Ravindran et al, 2018

SINGLE SOFT RESUMMATION

- SCALE IS Q^2/N_1
- RESUMMATION COEFFICIENTS ARE FUNCTIONS OF N_2
- HIGGS-LIKE CONTRIBUTION (SCALE Q^2/N_1^2) STARTS AT SUBLEADING ORDER

RAPIDITY DISTRIBUTIONS

SINGLE SOFT RESUMMATION (PRELIMINARY)

- SCALE IS Q^2/N_1
- RESUMMATION COEFFICIENTS ARE FUNCTIONS OF N_2

$$\hat{\sigma}(N_1, N_2, Q^2/\mu^2, \alpha_s(\mu^2)) = \sigma^{\rm LO}(\alpha_s(Q^2))g_0\left(\alpha_s(Q^2), N_2\right)$$
$$\times \exp \int_{Q^2}^{Q^2/N_1} \frac{d\mu^2}{\mu^2}\left[A[\alpha_s(\mu^2)]\ln\frac{Q^2/N_1}{\mu^2} + \bar{D}\left(\alpha_s^2(\mu^2), N_2\right)\right]$$
$$+\,\sigma^{\rm NLO}(\alpha_s(Q^2), N_1, N_2)\bar{g}_0\left(\alpha_s(Q^2), N_2\right)\exp\int_0^1 dx_1\,\frac{x^{N_1-1}-1}{1-x_1}\int_{\mu^2}^{Q^2(1-x_1)^2}\frac{dk^2}{k^2}A[\alpha_s(k^2)]$$

SCET RESULT

(Lustermans, Michel, Tackmann, 2019, unpublished, our dQCD translation)

$$\hat{\sigma}(N_1, N_2, Q^2/\mu^2, \alpha_s(\mu^2)) = \sigma^{\rm LO}(\alpha_s(Q^2))g_0\left(\alpha_s(Q^2), N_2\right)$$
$$\times \exp \int_{Q^2}^{Q^2/N_1} \frac{d\mu^2}{\mu^2}\left[A[\alpha_s(\mu^2)]\ln\frac{Q^2/N_1}{\mu^2} + \hat{\gamma}\left(\alpha_s(\mu^2), N_2\right) + \bar{\bar{D}}\left(\alpha_s^2(\mu^2), N_2\right)\right]$$

- $\hat{\gamma}(\alpha_s(\mu^2), N_2) = \gamma(\alpha_s(\mu^2), N_2) - A(\alpha_s(\mu^2))\ln\mu^2$: NONSINGULAR ANOMALOUS DIMENSIONS
- AGREEMENT UP TO NLL, BEYOND?

SUMMARY AND OUTLOOK

- RG APPROACH PROVIDES SIMPLE ALL-ORDER RESUMMATION, MUST MATCH TO FIXED ORDER
- EASILY GENERALIZED TO TWO SCALES
- TRANSVERSE MOMENTUM: SIMILAR TO INCLUSIVE, BUT TWO SCALES
- RAPIDITY DISTRIBUTION: DOUBLE SOFT OBTAINED FROM INCLUSIVE (SINGLE SCALE)
- RAPIDITY DISTRIBUTION, SINGLE SOFT: TWO SCALES