ASYMPTOTIC FREEDOM

IN
PARTON LANGUAGE

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN

The interdisciplinary contributions
of Giorgio Parisi to theoretical physics

Roma, 1 December 2022

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 740006

THE BIRTH

OF

PERTURBATIVE QCD

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

Istituto Nazionale di Fisica Nucleare

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of Giorgio Parisi to theoretical physics

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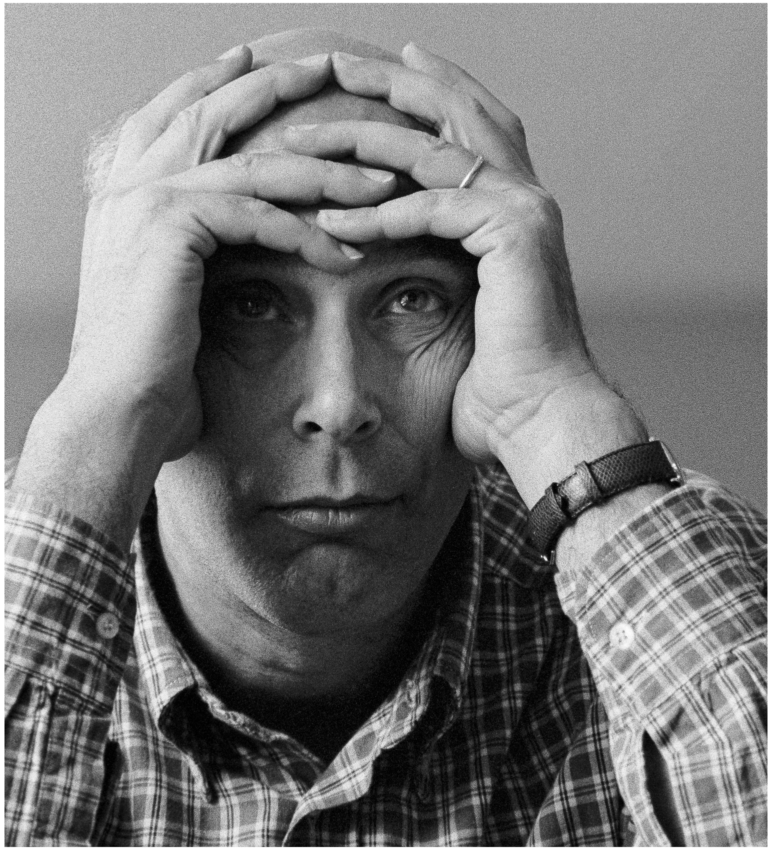
DISCLAIMER



Guido Altarelli (1941-2015)

- MY OWN UNDERSTANDING
- PERSONAL "HISTORICAL" RECONSTRUCTION & SELECTION

GIORGIO PARISI AND PERTURBATIVE QCD

- about 40 papers (out of more than 100) 1970-1980
- 1970-1973: Bjorken scaling and the parton model
- 1972-1980: scaling violations and perturbative QCD

SCALING AND THE PARTON MODEL

Generating functionals, ward identities and scalar mesons
G. Parisi (Rome U.), M. Testa (Rome U.) (1970)
Published in: *Nuovo Cim.A* 67 (1970) 13-22
DOI cite claim reference search 18 citations

Gauge invariance and dynamical symmetry breaking
G. Parisi (Rome U.), M. Testa (Rome U.) (1970)
Published in: *Lett.Nuovo Cim.* 4S1 (1970) 71-72, *Lett.Nuovo Cim.* 4 (1970) 71-72
DOI cite claim reference search 1 citation

Deep inelastic scattering and the nature of partons
N. Cabibbo (Rome U.), G. Parisi (Rome U.), M. Testa (Rome U.), A. Verganelakis (Democritos Nucl. Res. Ctr.) (1970)
Published in: *Lett.Nuovo Cim.* 4S1 (1970) 569-574, *Lett.Nuovo Cim.* 4 (1970) 569-574
DOI cite claim reference search 18 citations

Hadron Production in e+ e- Collisions
N. Cabibbo (Rome U.), G. Parisi (Rome U.), M. Testa (Rome U.) (Jul, 1970)
Published in: *Lett.Nuovo Cim.* 4S1 (1970) 35-39, *Lett.Nuovo Cim.* 4 (1970) 35-39
DOI cite claim reference search 153 citatio

Broken scale invariance, u(3) x u(3) breaking and hadron lagrangian structure
G. Parisi (Rome U.), M. Testa (Rome U.) (1971)
Published in: *Lett.Nuovo Cim.* 1S2 (1971) 549-552, *Lett.Nuovo Cim.* 1 (1971) 549-552
DOI cite claim reference search 0 citations

E+ e- annihilation into hadrons in the preasymptotic region and mu+ mu- production in proton collisions
G. Parisi (Frascati) (1971)
Published in: *Lett.Nuovo Cim.* 1S2 (1971) 1023-1025, *Lett.Nuovo Cim.* 1 (1971) 1023-1025

Hard bremsstrahlung in e+ e- collisions
G. Parisi (Frascati), F. Zirilli (Rome U.) (1971)
Published in: *Lett.Nuovo Cim.* 2S2 (1971) 395-396, *Lett.Nuovo Cim.* 2 (1971) 395-396
DOI cite claim reference search 0 citations

Angular correlations of the decay products of two heavy leptons
G. Parisi (Frascati), F. Zirilli (Rome U.) (1971)
Published in: *Lett.Nuovo Cim.* 2S2 (1971) 775-776, *Lett.Nuovo Cim.* 2 (1971) 775-776
DOI cite claim reference search 4 citations

The sigma-term and the scale breaking
G. Parisi (Frascati), M. Testa (Rome U.) (1971)
Published in: *Lett.Nuovo Cim.* 2S2 (1971) 1154-1156, *Lett.Nuovo Cim.* 2 (1971) 1154-1156
DOI cite claim reference search 0 citations

THE MEAN MASS OF THE BARYON OCTET AND BROKEN SCALE INVARIANCE
G. Parisi (Frascati), M. Testa (Rome U.) (Jan, 1971)
cite claim reference search 0 citations

SOME OBSERVATIONS ON THE PRODUCTION OF HADRONS THROUGH LANDAU LIFSHITZ PROCESSSES
G. Parisi (Frascati), M. Testa (Rome U.) (Mar, 1971)
cite claim reference search 0 citations

CALCULATION OF CRITICAL INDICES
G. Parisi (Frascati), L. Peliti (Rome U.) (Jun, 1971)
cite claim reference search 0 citations

THEORETICAL PREDICTIONS FOR CRITICAL EXPONENTS AT THE lambda POINT OF BOSE LIQUIDS
M. D'Eramo (Rome U.), G. Parisi (Frascati), L. Peliti (Rome U.) (Oct 23, 1971)
Published in: *Lett.Nuovo Cim.* 2 (1971) 17, 878-880
DOI cite claim reference search 123 citations

THE DYNAMICS OF CONFORMAL FIELD THEORIES: 1. EXACT INVARIANCE
G. Parisi (Frascati) (Nov, 1971)
cite claim reference search 0 citations

Conformal covariant correlation functions
S. Ferrara (Frascati), G. Parisi (Frascati) (1972)
Published in: *Nucl.Phys.B* 42 (1972) 281-290
DOI cite claim reference search 82 citations

Conformal invariance in perturbation theory
G. Parisi (Frascati) (1972)
Published in: *Phys.Lett.B* 39 (1972) 643-645
DOI cite claim reference search 82 citations

Canonical scaling and conformal invariance
S. Ferrara (Frascati), A.F. Grillo (Frascati), G. Parisi (Frascati), Raoul Gatto (Rome U.) (1972)
Published in: *Phys.Lett.B* 38 (1972) 333-334
DOI cite claim reference search 51 citations

A duality sum rule for deep inelastic scattering
G. Parisi (Frascati) (1972)
Published in: *Lett.Nuovo Cim.* 3S2 (1972) 395-396, *Lett.Nuovo Cim.* 3 (1972) 395-396

The shadow operator formalism for conformal algebra. Vacuum expectation values and operator products
S. Ferrara (Frascati), A.F. Grillo (Frascati), G. Parisi (Frascati), R. Gatto (Rome U.) (1972)
Published in: *Lett.Nuovo Cim.* 4S2 (1972) 115-120, *Lett.Nuovo Cim.* 4 (1972) 115-120
DOI cite claim reference search 150 citations

On self-consistency conditions in conformal covariant field theory
G. Parisi (Frascati) (1972)
Published in: *Lett.Nuovo Cim.* 4 (1972) 777-780
DOI cite claim reference search 38 citations

Nonequivalence between conformal covariant wilson expansion in euclidean and minkowski space
S. Ferrara (Frascati), A.F. Grillo (Frascati), G. Parisi (Frascati) (1972)
Published in: *Lett.Nuovo Cim.* 5S2 (1972) 147-151, *Lett.Nuovo Cim.* 5 (1972) 147-151
DOI cite claim reference search 47 citations

A simple method for computing electrodynamic processes of high order
G. Parisi (Frascati), F. Zirilli (Rome U.) (1972)
Published in: *Nuovo Cim.A* 11 (1972) 37-44
DOI cite claim reference search 1 citation

Conformal symmetry at lightlike distances and asymptotic behaviour of electromagnetic form factors
S. Ferrara (Frascati), A.F. Grillo (Frascati), G. Parisi (Frascati) (1972)
Published in: *Nuovo Cim.A* 12 (1972) 952-958
DOI cite claim reference search 9 citations

Covariant expansion of the conformal four-point function
S. Ferrara (Frascati), A.F. Grillo (Frascati), G. Parisi (Frascati), Raoul Gatto (Rome U.) (1972)
Published in: *Nucl.Phys.B* 49 (1972) 77-98, *Nucl.Phys.B* 53 (1973) 643-643 (erratum)
DOI cite claim reference search 195 citations

Bjorken scaling and the parton model
G. Parisi (Frascati) (1972)
Published in: *Phys.Lett.B* 42 (1972) 114-116
DOI cite claim reference search 8 citations

Critical indices for the spherical model from conformal covariant self consistency conditions

SCALING AND THE PARTON MODEL

SOME COMMON THEMES:

- PHENOMENOLOGICAL CONSEQUENCES OF SCALING
- WHAT IS THE NATURE OF PARTONS?
- EXPLAINING SCALING: CONFORMAL INVARIANCE?
- PHENOMENOLOGY: COMPUTATIONAL TECHNIQUES

SCALING AND THE PARTON MODEL

THE R RATIO

Hadron Production in e^+e^- Collisions (*).

N. CABIBBO

Istituto di Fisica dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

G. PARISI and M. TESTA
Istituto di Fisica dell'Università - Roma

(ricevuto il 30 Maggio 1970)

A DERIVATION OF THE (MODERN) LO RESULT:

This treatment leads to an asymptotic (very high cross-section c.m. energy, $2E$) of the form

$$\sigma \to \frac{\pi\alpha^2}{12E^2}\left[\sum_{\text{spin }0}(Q_i)^2 + 4\sum_{\text{spin }\frac{1}{2}}(Q_i)^2\right], \tag{1}$$

THE "STANDARD" DERIVATION

2. – The total cross-section for hadron production is proportional [8] to the absorptive part of the two-point correlation function of the e.m. current

$$\sigma = \frac{\alpha^2 4\pi^3}{E^2} \Pi(4E^2). \tag{2}$$

where

$$\Pi(q^2)(q_\mu q_\nu - q^2 \delta_{\mu\nu}) = (2\pi)^3 \sum_n \delta^4(p_n - q) \langle 0|I_\mu(0)|n\rangle \langle n|I_\nu(0)|0\rangle . \tag{3}$$

BUT NO WILSON EXPANSION $\Rightarrow$ "KINEMATIC" ARGUMENT

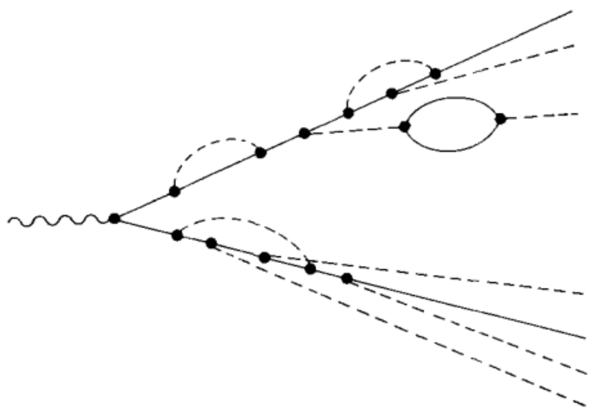


Fig. 1.

$\Rightarrow$ JUST EVALUATE M.E. OF FREE CURRENTS

WHAT ARE THE PARTONS?

Deep Inelastic Scattering and the Nature of Partons (*).

N. CABIBBO

Istituto di Fisica dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

G. PARISI and M. TESTA

Istituto di Fisica dell'Università - Roma

A. VERGANELAKIS

Nuclear Research Center « Democritos » - Athens

(ricevuto il 20 Giugno 1970)

1. – The results of inelastic-electron-scattering experiments have suggested the existence of pointlike constituents of the hadrons which have been named « partons » by FEYNMAN ([1]). Two alternatives are possible:

1) Partons are identified with quarks or other mythical components of the hadron.

2) Partons are associated with some of the usual hadrons.

PARTONS AS HADRONS

"...deep-inelastic scattering would not really investigate the structure of the proton, but that of the vacuum"

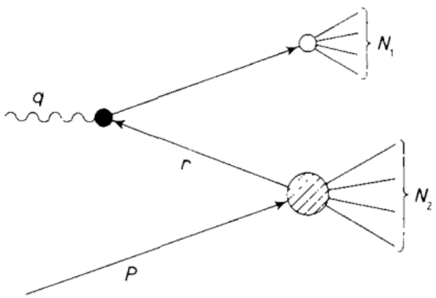


Fig. 1.

$\rightarrow$ MESONS AND BARYONS AS PARTONS $\Rightarrow$ PREDICT $\frac{\sigma_{\mathrm{long}}}{\sigma_{\mathrm{tranv}}}$

PARTON MODEL AND CURRENT ALGEBRA

THE "GOTTFRIED" SUM RULE

A Duality Sum Rule for Deep Inelastic Scattering.

G. PARISI

Laboratori Nazionali del CNEN - Frascati

(ricevuto il 23 Dicembre 1971)

$$\int_0^1 [F_{2P}(\omega) - F_{2N}(\omega)] \frac{d\omega}{\omega} = \frac{1}{3}\,.$$

(2) $$F^{ik}_{2B}(\omega) = \varepsilon(\omega)[\varrho(\omega)\,\delta_{ik} + D(\omega)\,d_{ik8} \pm d(\omega)\,d_{ik3}] + F(\omega)\,f_{ik8} \pm f(\omega)\,f_{ik3}\,,$$

where we must take the sign + for the proton and — for the neutron.

ASSUME t-CHANNEL RESONANCE EXCHANGE:

SR DERIVED USING SU(3)

THE MODERN DERIVATION (LO QCD):

$$\int_0^1 \frac{dx}{x}[F_2^p(x) - F_2^n(x)] = \int_0^1 dx\left[\frac{4}{9}(u(x)+\bar u(x)) + \frac{1}{9}(d(x)+\bar d(x)\right]_{p-n}$$

$$= \int_0^1 dx\,\frac{1}{3}\left[(u(x)+\bar u(x)) - (d(x)+\bar d(x))\right]_p \approx \int_0^1 dx\,\frac{1}{3}\left[(u(x)-\bar u(x)) - (d(x)-\bar d(x))\right]_p = \frac{1}{3}$$

USING ISOSPIN $u^p = d^n$, $u^n = d^p$ & THE APPROX $\bar u \approx \bar d$

WHY SCALING?

CONFORMAL INVARIANCE

BJORKEN SCALING AND THE PARTON MODEL

G. PARISI*
Laboratori Nazionali del C.N.E.N., Frascati, Italy

Received 17 July 1972

ASSUMPTIONS:

- $\beta(g_c) = 0,\ g_c \neq 0 \Rightarrow$ NONTRIVIAL FIXED POINT
- WILSON EXPANSION EXPANSION EXISTS ON THE LIGHT CONE, MASS-INDEP. LEADING TERM
- CONFORMAL INVARIANCE AT SHORT DISTANCES

PROOF:

- CANONICAL LIGHT-CONE SINGULARITIES IN PRODUCT OF TWO CURRENTS, MASSIVE $\Rightarrow$ ZERO-MASS THEORY IS FREE
- BJORKEN SCALING $\Rightarrow$ CANONICAL LIGHT CONE SINGULARITIES
- BJORKEN SCALING $\Rightarrow$ PARTON MODEL INVARIANCE AT SHORT DISTANCES

COMPUTATIONAL METHODS

$e^+e^- \to e^+e^-\gamma$ IN QED

Hard Bremsstrahlung in e⁺e⁻ Collisions.

G. PARISI
Laboratori Nazionali del CNEN - Frascati

F. ZIRILLI
Istituto di Matematica e Istituto di Fisica dell'Università - Roma

(ricevuto il 30 Giugno 1971)

- RESULT OBTAINED USING THE WEIZSÄCKER-WILLIAMS APPROXIMATION

This result is brought out in the following way: using an almost real approximation, one gets for the differential cross-section, in the case of zero-mass electrons,

$$\text{(1)} \qquad d\sigma = \sigma(\bar{\theta})\, d\Omega_e \frac{\alpha}{4\pi^2}\left[\frac{1}{\sin^2\theta_{in}} + \frac{1}{\sin^2\theta_{out}}\right]\frac{1+\eta^2}{\eta}\, d\Omega_\gamma d\eta\,,$$

where

$$\text{(2)} \qquad \sigma(\bar{\theta}) = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)} + \frac{1+\cos^2\theta}{2}\right]$$

is the differential cross-section for the Bhabha scattering.

- COMPARED TO EXACT RESULT & FOUND TO AGREE REASONABLY EVEN FOR LARGE ANGLE

COMPUTATIONAL METHODS

$e^+e^- \to e^+e^-\gamma$ IN QED

METHOD FOR THE EXACT COMPUTATION

A Simple Method for Computing Electrodynamic Processes of High Order.

G. Parisi

Laboratori Nazionali di Frascati del CNEN - Frascati

F. Zirilli

Istituto di Matematica e di Fisica dell'Università - Roma

(ricevuto l'11 Novembre 1971)

HELICITY TECHNIQUES:

1. – The basic relations.

As we said before we find the helicity amplitudes directly. The main simplification involved is due to the fact that if the number of the Feynman diagrams which contribute is N, there are N terms in the helicity amplitudes, but N^2 in the usual expression for $|A|^2$.

BUILDING BLOCKS

$$
\left\{
\begin{aligned}
&\overline{u}^+(\bar{\theta}, \bar{\varphi}, \bar{E})u^+(\theta, \varphi, E) = \sigma_0^{++}(\bar{\theta}, \bar{\varphi}; \theta, \varphi)[\alpha_1^+(\bar{E})a_1^+(E) - \alpha_2^+(\bar{E})\alpha_2^+(E)]\,,\\
&\overline{u}^+\gamma_0 u^+ \quad = \sigma_0^{++}(\bar{\alpha}_1^+\alpha_1^+ + \bar{\alpha}_2^+\alpha_2^+)\,,\\
&\overline{u}^+\gamma_k u^+ \quad = \sigma_k^{++}[\bar{\alpha}_1^+\alpha_2^+ + \bar{\alpha}_2^+\alpha_1^+]\,,\\
&\overline{u}^+\gamma_5 u^+ \quad = \sigma_0^{++}[\bar{\alpha}_1^+\alpha_2^+ - \bar{\alpha}_2^+\alpha_1^+]\,,\\
&\overline{u}^+\gamma_5\gamma_0 u^+ = \sigma_0^{++}[-\bar{\alpha}_2^+\alpha_1^+ - \bar{\alpha}_1^+\alpha_2^+]\,,\\
&\overline{u}^+\gamma_5\gamma_k u^+ = i\sigma_k^{++}[-\bar{\alpha}_2^+\alpha_1^+ - \bar{\alpha}_1^+\alpha_2^+]\,,\\
&\overline{u}^+\sigma_{0k} u^+ \quad = i\sigma_k^{++}[\bar{\alpha}_1^+\alpha_2^+ - \bar{\alpha}_2^+\alpha_1^+]\,,\\
&\overline{u}^+\sigma_{ik} u^+ \quad = \varepsilon_{ike}\sigma_e^{++}(\bar{\alpha}_1^+\alpha_1^+ - \bar{\alpha}_2^+\alpha_2^+)\,.
\end{aligned}
\right.
\tag{5}
$$

SCALING VIOLATIONS AND QCD PHENOMENOLOGY

Charmed Quarks and Asymptotic Freedom in Neutrino Scattering
Guido Altarelli (Rome U. and INFN, Rome), G. Parisi (Frascati), R. Petronzio (Rome U. and INFN, Rome) (Feb, 1976)
Published in: Phys.Lett.B 63 (1976) 183-187
DOI cite claim reference search 81 citations

On the Breaking of Bjorken Scaling
G. Parisi (Frascati), R. Petronzio (Rome U. and INFN, Rome) (Feb, 1976)
Published in: Phys.Lett.B 62 (1976) 331-334
DOI cite claim reference search 189 citations

A Mechanism for Confinement in Four-Dimensional Yang-Mills Theory
G. Parisi (Frascati) (Mar, 1976)
pdf cite claim reference search 0 citations

An Introduction to Scaling Violations
G. Parisi (Frascati) (Apr, 1976)
Contribution to: 11th Rencontres de Moriond, 83-114
pdf cite claim reference search 1 citation

Expanding Disk as a Dynamical Vacuum Instability in Reggeon Field Theory
D. Amati (CERN), G. Marchesini (CERN), M. Ciafaloni (Pisa, Scuola Normale Superiore and INFN, Pisa), G. Parisi (Frascati) (Jun, 1976)
Published in: Nucl.Phys.B 114 (1976) 483
pdf DOI cite claim reference search 64 citations

Mass Dependent Corrections to the Bjorken Scaling Law
R.Keith Ellis (Rome U. and INFN, Rome), R. Petronzio (Rome U. and INFN, Rome), G. Parisi (Frascati) (Jun, 1976)
Published in: Phys.Lett.B 64 (1976) 97-101
DOI cite claim reference search 67 citations

Asymptotic Estimates in Perturbation Theory
G. Parisi (IHES, Bures-sur-Yvette) (Dec, 1976)
Published in: Phys.Lett.B 66 (1977) 167-169
DOI cite claim reference search 86 citations

Asymptotic Estimates in Scalar Electrodynamics

Asymptotic Estimates of Feynman Diagrams
G. Parisi (IHES, Bures-sur-Yvette) (1977)
Published in: Phys.Lett.B 68 (1977) 361-364
DOI cite claim reference search 24 citations

The Perturbative Expansion and the Infinite Coupling Limit
G. Parisi (IHES, Bures-sur-Yvette) (1977)
Published in: Phys.Lett.B 69 (1977) 329-331
DOI cite claim reference search 33 citations

Perturbation Theory at Large Orders for Potential with Degenerate Minima
E. Brezin (Saclay), G. Parisi (IHES, Bures-sur-Yvette), Jean Zinn-Justin (Saclay) (Feb, 1977)
Published in: Phys.Rev.D 16 (1977) 408-412
DOI cite claim reference search 113 citations

Asymptotic Freedom in Parton Language
Guido Altarelli (Ecole Normale Superieure), G. Parisi (IHES, Bures-sur-Yvette) (Mar, 1977)
Published in: Nucl.Phys.B 126 (1977) 298-318
DOI cite claim reference search 7,819 citations

Asymptotic Estimates in Quantum Electrodynamics
C. Itzykson (Saclay), G. Parisi (IHES, Bures-sur-Yvette), J.-B. Zuber (Saclay) (Mar, 1977)
Published in: Phys.Rev.D 16 (1977) 996-1013
DOI cite claim reference search 82 citations

Asymptotic Estimates in Quantum Electrodynamics. 2.
R. Balian (Saclay), C. Itzykson (Saclay), G. Parisi (IHES, Bures-sur-Yvette), J.B. Zuber (Saclay) (Aug, 1977)
Published in: Phys.Rev.D 17 (1978) 1041-1052
DOI cite claim reference search 50 citations

The Physical Basis of the Asymptotic Estimates in Perturbation Theory
G. Parisi (Ecole Normale Superieure) (Sep, 1977)
Contribution to: Cargese Summer Institute: Hadron Structure and Lepton-Hadron Interactions, 665
pdf cite claim reference search 0 citations

Transverse Momentum in Drell-Yan Processes
Guido Altarelli (Rome U. and INFN, Rome), G. Parisi (Ecole Normale Superieure), R. Petronzio (CERN) (Oct, 1977)
Published in: Phys.Lett.B 76 (1978) 351-355
DOI cite claim reference search 223 citations

Planar Diagrams
E. Brezin (Saclay), C. Itzykson (Saclay), G. Parisi (Saclay), J.B. Zuber (Saclay) (Nov, 1977)
Published in: Commun.Math.Phys. 59 (1978) 35
DOI cite claim reference search 1,415 citations

Bounds on the Number and Masses of Quarks and Leptons
L. Maiani (Ecole Normale Superieure), G. Parisi (Ecole Normale Superieure), R. Petronzio (CERN) (Dec, 1977)
Published in: Nucl.Phys.B 136 (1978) 115-124
DOI cite claim reference search 360 citations

Super Inclusive Cross-Sections
G. Parisi (Ecole Normale Superieure) (Jan, 1978)
Published in: Phys.Lett.B 74 (1978) 65-67
pdf DOI cite claim reference search 218 citations

Transverse Momentum of Muon Pairs Produced in Hadronic Collisions
Guido Altarelli (Rome U. and INFN, Rome), G. Parisi (Ecole Normale Superieure), R. Petronzio (CERN) (Jan, 1978)
Published in: Phys.Lett.B 76 (1978) 356-360
DOI cite claim reference search 182 citations

Singularities of the Borel Transform in Renormalizable Theories
G. Parisi (Ecole Normale Superieure) (Mar, 1978)
Published in: Phys.Lett.B 76 (1978) 65-66
pdf DOI cite claim reference search 207 citations

TRACE IDENTITIES FOR THE SCHRODINGER OPERATOR AND THE WKB METHOD
G. Parisi (Ecole Normale Superieure) (Mar, 1978)
pdf cite claim reference search 0 citations

On Infrared Divergences
G. Parisi (Ecole Normale Superieure) (Apr, 1978)
Published in: Nucl.Phys.B 150 (1979) 163-172
pdf DOI cite claim reference search 191 citations

SCALING VIOLATIONS AND QCD PHENOMENOLOGY

SOME COMMON THEMES:

- ANOMALOUS DIMENSIONS AND SCALE DEPENDENCE
- PHENOMENOLOGY: PARTONS VS. OPERATORS
- PERTURBATIVE QCD PHENOMENOLOGY
- ALL-ORDER BEHAVIOUR & RESUMMATION

SCALING VIOLATIONS OF STRUCTURE FUNCTIONS

EXPERIMENTAL LIMITS ON THE VALUES OF ANOMALOUS DIMENSIONS

G. PARISI
Laboratori Nazionali del C.N.E.N., Frascati, Italia

Received 31 October 1972

It is known that Bjorken [1] scaling is violated in perturbation theory, because of the appearance of annoying $\log(q^2)$ [e.g. 2]. This fact may be completely

MELLIN MOMENTS AND ANOMALOUS DIMENSIONS:

$$\int_1^\infty F_2(\omega, q^2)\, \omega^{-N} \mathrm{d}\omega \to \sum_i C_N^i \left(\frac{M^2}{q^2}\right)^{\sigma_N^i/2},$$

$N = 2, 4, \ldots,$

$$\sigma_N = A\left[1 - \frac{12}{(N+1)(N+2)}\right].$$

$$P(x) = A[\delta(1-x) - x(1-x)]$$

APPROXIMATE SCALING VIOLATION AS A CONVOLUTION

APPROXIMATE SCALING FOR $x \sim 0.1$

The new "scaling law", which may be obtained expanding the right hand side of eq. (1) in powers of $\log(q^2/K^2)$ and using the Faltung-theorem for the Mellin transforms*, is:

$$F_2(\omega, q^2) = F_2(\omega, K^2)\left[1 - \Delta(\omega, K^2)\frac{A}{2}\log\frac{q^2}{K^2} + O\left(A^2 \log\frac{q^2}{K^2}\right)\right],$$

$$\Delta(\omega, q^2) = +1 - \int_{1/\omega}^{1} \mathrm{d}x (x^2 - x^3)\, \frac{F_2(\omega x, q^2)}{F_2(\omega, q^2)}. \qquad (3)$$

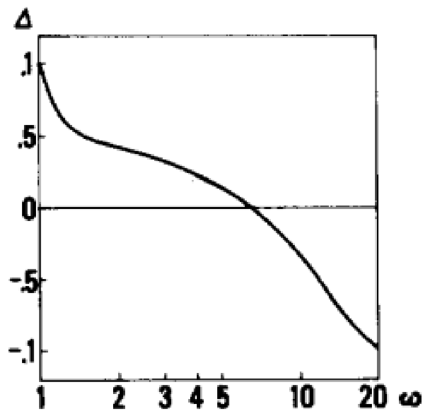


Fig. 1. Plot of the function $\Delta(\omega)$.

80 5 *Applying QCD to deep inelastic scattering*

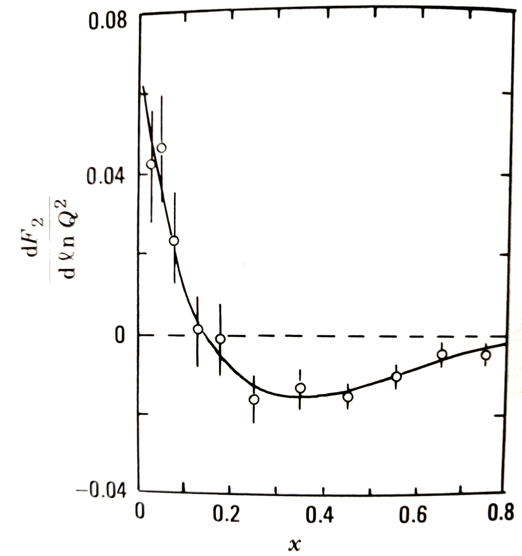


Fig. 5.7 Derivative of the structure function F_2 measured in muon-proton scattering by EMC (Aubert *et al.* 1986). The curve corresponds to a leading order fit with $\Lambda = 90$ MeV.

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SCALING VIOLATIONS: THEORY AND PHENOMENOLOGY

- CAN ONE COMPUTE THE ANOMALOUS DIMENSIONS?

HOW TO MEASURE THE DIMENSION OF THE PARTON FIELD

G. PARISI
Laboratori Nazionali di Frascati del CNEN, Frascati, Italy

Received 15 January 1973

Abstract: We show that the anomalous dimension of the fundamental field is connected to the anomalous dimensions of the high spin bilinear operators. The dimensions of these operators can be determined by examining the violations of the Bjorken scaling law in deep-inelastic

- CAN ONE MEASURE THE ANOMALOUS DIMENSIONS?

GROSS-WILCZEK-POLITZER ANOMALOUS DIMENSIONS

CHARMED QUARKS AND ASYMPTOTIC FREEDOM IN NEUTRINO SCATTERING

G. ALTARELLI, R. PETRONZIO
Istituto di Fisica, Roma, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy
and
G. PARISI
Istituto Nazionale di Fisica Nucleare, Frascati, Italy

Received 24 March 1976

Asymptotic freedom and charm production are both important ingredients for a theoretical analysis of neutrino cross sections. We study in detail the Q^2 dependence of integrated quantities like cross sections, y-distributions and $\langle x \rangle$ values. Deviations from scaling are quite substantial in the present energy range.

ON THE BREAKING OF BJORKEN SCALING

G. PARISI
Istituto Nazionale di Fisica Nucleare, Frascati (Roma), Italy
and
R. PETRONZIO
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In the framework of coloured quark model we obtain detailed predictions for the q^2 dependence of the structure functions of the proton and the neutron and the $\sigma_{\bar{\nu}}/\sigma_\nu$ ratio

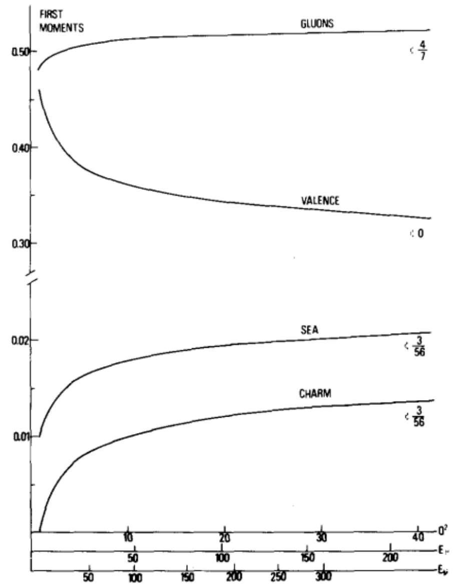

Fig. 1. $\int_0^1 x f(x)\,dx$ for $f(x) \equiv G(x)$ (gluons), $V(x)$ (valence), $s(x)$ (SU(3) symmetric sea), $c(x)$ (charm) as functions of Q^2 (or E_ν and $E_{\bar{\nu}}$). The arrows indicate the asymptotic values. The curves are computed from eqs. (3) with the initial values eq. (6) which correspond to $A_8^{(1)} = 0.46$, $A_{15}^{(1)} = 0.52$, $A_S^{(1)} = 0.16$.

THE SCALE DEPENDENCE OF STRUCTURE FUNCTIONS

THE ALTARELLI-PARISI EQUATION! (1974):

DETAILED PREDICTIONS FOR THE p-n STRUCTURE FUNCTIONS IN THEORIES WITH COMPUTABLE LARGE MOMENTA BEHAVIOUR

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If strong interactions are described by non Abelian gauge theories, detailed predictions are obtained for the q^2 dependence at fixed ω of the difference of the structure functions of the proton and neutron.

Different powers behaviour comes from the contributions of different operators. However in the $I = 1$ t-channel only one operator is present; a simple relation is satisfied by the difference between the proton and the neutron structure functions [6]:

$$\int_1^\infty F^2_{\mathrm{p-n}}(\omega, q^2)\, \omega^{-N}\, \mathrm{d}\omega \xrightarrow[q^2\to\infty]{} C^N_{\mathrm{p-n}} (\log q^2)^{-A^N}$$
$$A^N = \tfrac{2}{27}\{1 - 2/N(N+1) + 4\,[\psi(N+1) + \gamma^{-1}]\} \qquad (2)$$

where ψ is the logaritmical derivative of the Euler Γ function and γ is the Euler-Mascheroni constant.

A very nice test of the validity of the theoretical scheme would be provided by a comparison of (2) with the experimental data.

The aim of this note is to point out that it is possible to derive from (2) simple consequences on the q^2 behaviour of the structure functions at fixed ω. The final result is:

$$D(\omega, q^2) \equiv q^2 \log q^2 \frac{\partial}{\partial q^2} F^2_{\mathrm{p-n}}(\omega, q^2) \qquad (3)$$
$$= \tfrac{2}{27}\left\{[3 + 4\log(1 - 1/\omega)]\, F^2_{\mathrm{p-n}}(\omega, q^2) + \int_1^\omega \frac{\mathrm{d}\mu}{\mu^2}\left[\left(2 - \frac{2}{\mu} + \frac{4}{\mu-1}\right) F^2_{\mathrm{p-n}}\left(\frac{\omega}{\mu}, q^2\right) - \frac{4\mu}{\mu-1} F^2_{\mathrm{p-n}}\left(\omega, q^2\right)\right]\right\}.$$

Eq. (2) and eq. (3) are mathematically equivalent, however we believe that the second equation is easier to test than the first one. A salient feature of eq. (3) is that the integral in its r.h.s. is done from 1 up to ω: informations on the high ω behaviour of the structure functions are not needed.

PROOF: (1) FOLLOWS FROM A RGE, TAKE INVERSE MELLIN

THE ALTARELLI-PARISI EQUATION

A PREVIEW: THE MORIOND LECTURES (APRIL 1976)

AN INTRODUCTION TO SCALING VIOLATIONS

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Abstract: The theory of scaling violations in deep inelastic scattering is presented using the parton model language; intuitive physical arguments are used as far as possible. In the comparison between theory and experiments particular attention is payed to the consequences of the opening of the threshold for charm production.

Part of the results presented here have been obtained by the author in collaboration with

G. Altarelli and R. Petronzio

- "THE CONSTITUENTS OF THE ELECTRON": PHOTON SHOWERING IN QED $\Rightarrow$ "GRIBOV-LIPATOV" EQUATIONS

However the situation is not so simple: the photon itself may split in a $e\bar{e}$ pair, each of the new born e or $\bar{e}$ may emit a photon and so on. The whole process is quite similar to the evolution of an electromagnetic shower in lead. A typical diagram is shown in Fig. 2.

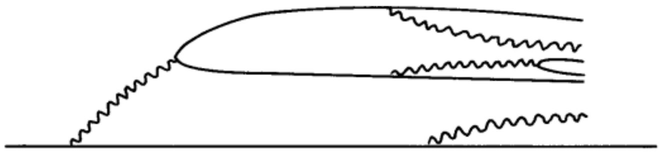

fig. 2

FIG. 2 - A typical diagram contributing to the formation of the "shower".

It is clear that we must introduce in the game the distributions of the e, $\bar{e}$ and γ inside the electron; using the same arguments as in the previous case a more complicated master equation can be derived:

$$\frac{dN_e(x,L)}{dL} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left[N_e(y,L)\,p_{ee}(x/y) + N_\gamma(y,L)\,p_{e\gamma}(x/y)\right],$$

$$\frac{dN_{\bar{e}}(x,L)}{dL} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left[N_{\bar{e}}(y,L)\,p_{\bar{e}\bar{e}}(x/y) + N_\gamma(y,L)\,p_{\bar{e}\gamma}(x/y)\right], \qquad (3.12)$$

- INTUITIVE ARGUMENT FOR ASYMPTOTIC FREEDOM: ANTI-SCREENING "ENANTION"

- QCD EVOLUTION EQUATIONS WRITTEN IN FULL, BUT SPLITTING FUNCTIONS NOT DERIVED

- COMPARISON TO DATA

THE ALTARELLI-PARISI EQUATION

ASYMPTOTIC FREEDOM IN PARTON LANGUAGE

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Received 12 April 1977

A novel derivation of the Q^2 dependence of quark and gluon densities (of given helicity) as predicted by quantum chromodynamics is presented. The main body of predictions of the theory for deep-inleastic scattering on either unpolarized or polarized targets is re-obtained by a method which only makes use of the simplest tree diagrams and is entirely phrased in parton language with no reference to the conventional operator formalism.

In this paper we show that an alternative derivation of all results of current interest for the Q^2 behaviour of deep inelastic structure functions is possible. In this approach all stages of the calculation refer to parton concepts and offer a very illuminating physical interpretation of the scaling violations. In our opinion the present approach, although less general, is remarkably simpler than the usual one since all relevant results can be derived in a direct way from the basic vertices of QCD, with no loop calculations being involved (the only exception is the lowest order expression for the running coupling constant which we do not rederive).

This method can be described as an appropriate generalization of the equivalent photon approximation in quantum electrodynamics [8]. A preliminary and less complete version of this paper has been presented by one of us at the 1976 Flaine meeting [9].

MOMENTS $\Rightarrow$ RGE$\Rightarrow$ AP EQUATION

$$M_n^{\mathrm{NS}}(t) = \int_0^1 \mathrm{d}x\, x^{n-1}\, q^{\mathrm{NS}}(x, t)\,. \tag{11}$$

As is well known [4,5] the predicted t dependence of moments is of the form

$$M_n^{\mathrm{NS}}(t) = M_n^{\mathrm{NS}}(0) \left[\frac{\alpha}{\alpha(t)}\right]^{A_n^{\mathrm{NS}}/2\pi b}\,. \tag{12}$$

$\Rightarrow$

$$\frac{\mathrm{d}M_n^{\mathrm{NS}}(t)}{\mathrm{d}t} = \frac{\alpha(t)}{2\pi} A_n^{\mathrm{NS}}\, M_n^{\mathrm{NS}}(t)\,, \tag{17}$$

with assigned initial value $M_n^{\mathrm{NS}} = M_n^{\mathrm{NS}}(0)$ at $t = 0$. In turn, for any n (sufficiently large) the whole set of eqs. (17) is equivalent to the following master equation for the densities:

$$\frac{\mathrm{d}q^{\mathrm{NS}}(x, t)}{\mathrm{d}t} = \frac{\alpha(t)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y}\, q^{\mathrm{NS}}(y, t)\, P\!\left(\frac{x}{y}\right), \tag{18}$$

provides that

$$\int_0^1 \mathrm{d}z\, z^{n-1}\, P(z) = A_n^{\mathrm{NS}}\,. \tag{19}$$

AND ITS MEANING

It is convenient to rewrite eq. (18) in the form

$$q^{\mathrm{NS}}(x, t) + \mathrm{d}q^{\mathrm{NS}}(x, t) = \int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}z\, \delta(zy - x)\, q^{\mathrm{NS}}(y, t) \left[\delta(z-1) + \frac{\alpha}{2\pi} P(z)\, \mathrm{d}t\right]. \tag{20}$$

The meaning of this equation is clear. Given a quark with momentum y there is a chance that is radiates a gluon, thus reducing its energy from y to x. The quantity

$$\mathcal{P}_{\mathrm{qq}} + \mathrm{d}\mathcal{P}_{\mathrm{qq}} = \delta(z-1) + \frac{\alpha}{2\pi} P(z)\, \mathrm{d}t \tag{21}$$

is the probability density of finding, inside a quark, another quark with fraction z of the parent momentum. The change with t of this probability produces the variation of the quark distribution function. Thus $P(z)\, \alpha/2\pi$ is the variation per unit t at order α of the probability density of finding inside a quark another quark with fraction z of the parent momentum.

THE SPLITTING FUNCTION

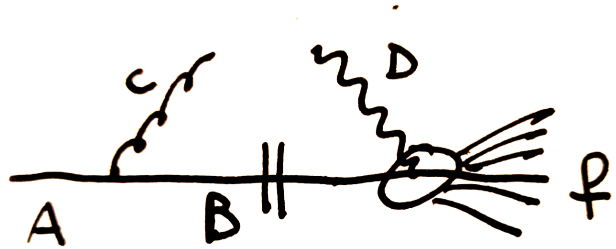
SPLITTING PROBABILITY & SPLITTING FUNCTION

$$\mathrm{d}\mathcal{P}_{\mathrm{BA}}(z)\,\mathrm{d}z = \frac{\alpha}{2\pi} P_{\mathrm{BA}}(z)\,\mathrm{d}z\,\mathrm{d}t\ .$$

THE WEIZSÄCKER-WILLIAMS APPROXIMATION

MATRIX ELEMENT

$$\frac{\not p_B}{p_B^2} = \sum_r u^r(p_B)\bar{u}^r(p_B) \text{ on shell;} \qquad \Rightarrow \qquad M_{\mathrm{A+D\to C+f}} = g^2 \frac{V_{\mathrm{A\to B+C}}\, V_{\mathrm{B+D\to f}}}{(2E_{\mathrm{B}})(E_{\mathrm{B}}+E_{\mathrm{C}}-E_{\mathrm{A}})}\,,$$



$$\mathrm{d}P_{\mathrm{BA}}\,\mathrm{d}z \quad \otimes \quad \mathrm{d}\sigma_f$$

CROSS SECTION

$$\mathrm{d}\sigma_a = \frac{g^4}{8E_{\mathrm{A}}E_{\mathrm{D}}} \frac{|V_{\mathrm{A\to B+C}}|^2 |V_{\mathrm{B+D\to f}}|^2}{(2E_{\mathrm{B}})^2(E_{\mathrm{B}}+E_{\mathrm{C}}-E_{\mathrm{A}})^2}$$

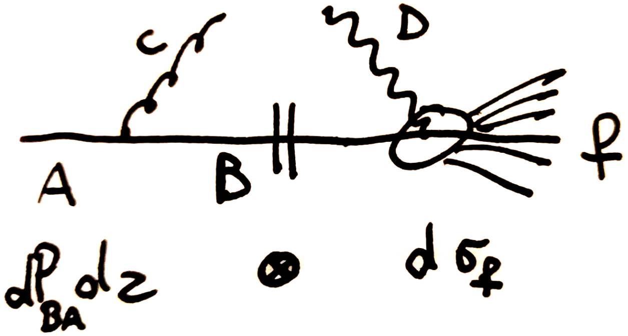
$$\times (2\pi)^2\,\delta^4(k_{\mathrm{A}}+k_{\mathrm{D}}-k_{\mathrm{C}}-k_{\mathrm{f}}) \frac{\mathrm{d}^3k_{\mathrm{C}}}{(2\pi)^3(2E_{\mathrm{C}})} \prod_f \frac{\mathrm{d}^3p_{\mathrm{f}}}{(2\pi)^3(2E_{\mathrm{f}})}\,,$$

SPLITTING PROBABILITY

$$\mathrm{d}\mathcal{P}_{\mathrm{BA}}(z)\mathrm{d}z = \frac{E_{\mathrm{B}}}{E_{\mathrm{A}}} \frac{g^2|V_{\mathrm{A\to B+C}}|^2}{(2E_{\mathrm{B}})^2(E_{\mathrm{B}}+E_{\mathrm{C}}-E_{\mathrm{A}})^2} \frac{\mathrm{d}^3k_{\mathrm{C}}}{(2\pi)^3(2E_{\mathrm{C}})}\,,$$

COMPUTING THE SPLITTING FUNCTION

THE SPLITTING PROBABILITY



$$d\mathcal{P}_{BA}(z)dz = \frac{E_B}{E_A}\frac{g^2|V_{A\to B+C}|^2}{(2E_B)^2(E_B+E_C-E_A)^2}\frac{d^3k_C}{(2\pi)^3(2E_C)},$$

KINEMATICS

$$k_A = (P; P, \mathbf{0}),$$

$$k_B = \left(zP + \frac{p_\perp^2}{2zP}; zP, \boldsymbol{p}_\perp\right),$$

$$k_C = \left((1-z)P + \frac{p_\perp^2}{2(1-z)P}; (1-z)P, -\boldsymbol{p}_\perp\right).$$

PHASE SPACE

$$(2E_B)^2(E_B + E_C - E_A)^2 = \frac{(p_\perp^2)^2}{(1-z)^2},$$

$$\frac{d^3k_C}{(2\pi)^3(2E_C)} = \frac{dz\, dp_\perp^2}{16\pi^2(1-z)}.$$

- "OLD" PERTURBATION THEORY
- ENERGY NOT CONSERVED
- CAN USE ON-SHELL SPINORS

SPLITTING PROBABILITY

$$d\mathcal{P}_{BA}(z) = \frac{\alpha}{2\pi}\frac{z(1-z)}{2}\overline{\sum_{\rm spins}}\frac{|V_{A\to B+C}|^2}{p_\perp^2}\, d\ln p_\perp^2,$$

COMPUTING THE SPLITTING FUNCTION

THE MATRIX ELEMENT

SPLITTING PROBABILITY

$$d\mathcal{P}_{BA}(z) = \frac{\alpha}{2\pi}\frac{z(1-z)}{2}\overline{\sum_{\text{spins}}}\frac{|V_{A\to B+C}|^2}{p_\perp^2}\, d\ln p_\perp^2 ,$$

SPLITTING FUNCTION

$$P_{BA}(z) = \tfrac{1}{2}\, z(1-z)\overline{\sum_{\text{spins}}}\frac{|V_{A\to B+C}|^2}{p_\perp^2} \qquad (z<1) ,$$

COLOR AND SPIN SUMS

$$C_2(\mathrm{R}) = \frac{1}{N}\sum_a t^a t^a = \frac{N^2-1}{2N}$$

MATRIX ELEMENT

$$\overline{\sum_{\text{spins}}} |V_{q\to Gq}|^2 = \tfrac{1}{2} C_2(\mathrm{R})\, \mathrm{Tr}(\not{k}_C \gamma_\mu \not{k}_A \gamma_\nu) \overline{\sum_{\text{pol}}} \epsilon^{*\mu}\epsilon^{\nu} ,$$

$$\sum_{\text{pol}} \epsilon^*\epsilon \to \delta^{ij} - \frac{k_B^i k_B^j}{k_B^2} \qquad (i,j = 1,2,3) .$$

THE SPLITTING FUNCTIONS

$$B \leftrightarrow C \qquad \Leftrightarrow \qquad P_{gq}(x) = P_{qq}(1-x)$$

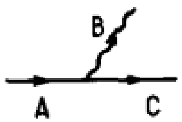


Fig. 2. The quark gluon vertex which determines P_{Gq} and P_{qq}. The form of the vertex is $ig\bar{q}_C\gamma^\mu t^a q_A B^a_\mu$ with $\mathrm{Tr}\, t^a t^b = \frac{1}{2}\delta^{ab}$.

$$P_{Gq}(z) = C_2(\mathrm{R})\frac{1+(1-z)^2}{z} , \qquad P_{qq}(z) = C_2(\mathrm{R})\frac{1+z^2}{1-z} \qquad (z<1)$$

SPLITTING FUNCTION AND ANOMALOUS DIMENSION

- COMPUTE MELLIN MOMENTS

- MOMENTS OF P_{qq}, P_{gg} DIVERGE AT $x = 1 \Rightarrow$ PLUS PRESCRIPTION

$$\int_0^1 \frac{\mathrm{d}z\, f(z)}{(1-z)_+} \equiv \int_0^1 \mathrm{d}z \frac{f(z) - f(1)}{1-z} = \int_0^1 \mathrm{d}z \ln(1-z) \frac{\mathrm{d}}{\mathrm{d}z} f(z)\,,$$

- $\int_0^1 x^{n-1} \delta(1-x) = 1 \Rightarrow$ ADDITIVE CONSTANT IN ANOMALOUS DIMENSION

- CONSTANT FIXED BY SUM RULES: $\int_0^1 dx P_{qq}(x) = 0$ CHARGE CONSERVATION;

$\int_0^1 dx x\,(P_{qg} + P_{gg}) = \int_0^1 dx x\,(P_{qq} + P_{gq}) = 0$ MOMENTUM CONSERVATION

THE ANOMALOUS DIMENSIONS

$$\int_0^1 \mathrm{d}z\, z^{n-1} P_{\mathrm{qq}}(z) \equiv A_n^{\mathrm{NS}} = C_2(\mathrm{R}) \left[-\tfrac{1}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=2}^{n} \frac{1}{j} \right],$$

$$\int_0^1 \mathrm{d}z\, z^{n-1} P_{\mathrm{Gq}}(z) \equiv A_n^{\mathrm{Gq}} = C_2(\mathrm{R}) \frac{2+n+n^2}{n(n^2-1)}\,,$$

$$2f \int_0^1 \mathrm{d}z\, z^{n-1} P_{\mathrm{qG}}(z) \equiv 4T(\mathrm{R}) A_n^{\mathrm{qG}} = 2T(\mathrm{R}) \frac{2+n+n^2}{n(n+1)(n+2)}\,,$$

$$\int_0^1 \mathrm{d}z\, z^{n-1} P_{\mathrm{GG}}(z) \equiv A_n^{\mathrm{GG}}$$

$$= C_2(\mathrm{G}) \left[-\frac{1}{6} + \frac{2}{n(n-1)} + \frac{2}{(n+1)(n+2)} - 2 \sum_{j=2}^{n} \frac{1}{j} - \frac{2}{3} \frac{T(\mathrm{R})}{C_2(\mathrm{G})} \right].$$

“This set of logarithmic exponents is seen to coincide with the results of Georgi, Politzer, Gross, Wilczek”.

FURTHER DEVELOPMENTS

- THE POLARIZED CASE!

$$|V_{G_+\to q_\pm\bar{q}}|^2 = \frac{1}{2}\,\mathrm{Tr}\left(\not{k}_C\gamma_\mu\not{k}_B\gamma_\nu\,\frac{1\pm\gamma_5}{2}\right)\epsilon_+^{*\mu}\epsilon_+^{\nu}$$

$$= \frac{p_\perp^2}{z(1-z)}\,[(z^2+(1-z)^2)\pm(z^2-(1-z)^2)]\ .$$

FULL SET OF POLARIZED SPLITTING FUNCTIONS

- BEYOND LEADING LOG:

RGE AND THE RUNNING OF α_s AT LEADING LOG

$$\left[\frac{\partial}{\partial t}-\beta(\alpha)\frac{\partial}{\partial\alpha}-\gamma_n(\alpha)\right]M_n(\alpha,t)=0\ . \qquad \left[\frac{\mathrm{d}}{\mathrm{d}t}-\gamma_n(\alpha(t))\right]M_n\approx\left[\frac{\mathrm{d}}{\mathrm{d}t}-\frac{\alpha(t)}{2\pi}A_n\right]M_n=0\ .$$

RUNNING $\alpha_s(t)$ RESUMS RGE AT LL ACCURACY

PHENOMENOLOGY BEYOND ALTARELLI-PARISI

TRANSVERSE MOMENTUM IN DRELL–YAN PROCESSES

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We study the transverse momentum distribution of muon pairs from Drell–Yan processes in QCD. In particular the dependence of $\langle k_\perp^2 \rangle$ on Q^2 is considered. QCD predicts an approximately linear rise of $\langle k_\perp^2 \rangle$ with S or Q^2 only at fixed $\tau = Q^2/S$. The slope as a function of τ is quantitatively studied for PP and P–nucleus scattering. The most recent data showing a rather flat $\langle k_\perp^2 \rangle$ in Q^2 at fixed S are found to be consistent with QCD.

TRANSVERSE MOMENTUM OF MUON PAIRS PRODUCED IN HADRONIC COLLISIONS

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Received 13 February 1978

The observed $p_\perp$ distributions of large mass muon pairs produced in proton–nucleus collisions are quantitatively explained in QCD by also explicitly taking into account the intrinsic wave function $p_\perp$ spread of partons inside the hadrons. The experimental data indicate $\langle p_\perp \rangle \sim$ 500–600 MeV for each parton.

A SIMPLE PARAMETRIZATION OF THE Q^2 DEPENDENCE OF THE QUARK DISTRIBUTIONS IN QCD

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Received 4 September 1978

We propose the following parametrization of the deep inelastic structure functions:

$$F(x, Q^2) = x^\beta (1-x)^{\alpha(Q^2)} \sum_{n=0}^{N} P_n^{(\alpha,\beta)}(x) f_n(Q^2)$$
$$= x^\beta (1-x)^{\alpha(Q^2)} \sum_{n=0}^{N} d(N, n, Q^2) x^n ,$$

where $P_n^{(\alpha,\beta)}(x)$ are Jacobi polynomials and where the Q^2 dependence of the $\alpha(Q^2)$ and $f_n(Q^2)$ coefficients is deduced from QCD. We show that with few terms in this expansion we get good accuracy in the 3 GeV$^2 < Q^2 <$ 15 000 GeV^2 range. We also give a very simple fit to the $d(N, n, Q^2)$ for the valence part of the structure function.

SUPERINCLUSIVE CROSS SECTIONS

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Received 23 January 1978

We define superinclusive cross sections; their value aims to give a global characterization of the structure of the final state. We point out that in many cases, superinclusive cross sections can be computed using the renormalization group.

GLUON FRAGMENTATION FUNCTIONS FROM QUARK JETS

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Received 1 December 1978

The analysis of events with a fixed, different from unity, fraction of energy flowing into a solid angle $\Delta\Omega$ can be used to extract the probability functions related to the scaling violations of deep inelastic scattering. One can obtain from the study of a quark jet, both the gluon and the quark fragmentation functions into hadrons.

TESTABLE QCD PREDICTIONS FOR SPHERICITY-LIKE DISTRIBUTIONS IN e^+e^- ANNIHILATION

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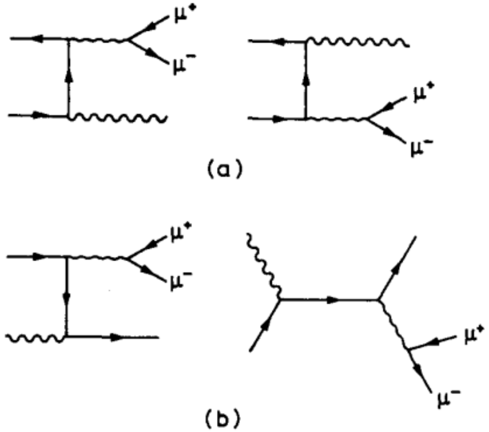
Received 30 October 1979

We give a procedure to define sphericity-type variables which are infrared finite and not very sensitive to jet pionization. QCD predictions for the distribution of these variables can be computed in perturbation theory. We give numerical results in some examples.

THE DRELL-YAN p_t DISTRIBUTION



$$\frac{d\sigma_{q\bar{q}}}{dQ^2 d\cos\theta} = \frac{8\alpha^2\alpha_s}{27Q^2} e^2 \frac{s-Q^2}{s^2\sin^2\theta} \left\{ 1+\cos^2\theta + 4\frac{Q^2 s}{(s-Q^2)^2} \right\},$$

$$\frac{d\sigma_{qG}}{dQ^2 d\cos\theta} = \frac{\alpha^2\alpha_s}{18Q^2} e^2 \frac{s-Q^2}{s^2(1+\cos\theta)} \left\{ \frac{2s}{s-Q^2} + \frac{s-Q^2}{2s}(1+\cos\theta)^2 - \frac{2Q^2}{s}(1-\cos\theta) \right\},$$

$\langle k_t^2 \rangle$ COMPUTED CONVOLUTING WITH PARTON LUMI

"With the advent of QCD factorization one was able to write the DY cross-section at measured Q_T" (Collins, Soper, Sterman, 1984)

THE JET SHAPE VARIABLES

An interesting example of a superinclusive quantity is given by the 3 by 3 matrix [1,2]:

$$\theta_{ij} = \sum_k \frac{p_i^{(k)} p_j^{(k)}}{|p^{(k)}|}, \quad i=1,2,3; \; j=1,2,3. \qquad (1)$$

$p_i^{(k)}$ are the spatial components of the momentum of the kth particle in the center-of-mass frame $[|p^{(k)}|^2 = \Sigma_i (p_i^{(k)})^2]$ and the sum runs over all the particles in the final state.

EIGENVECTORS DEFINE STANDARD SHAPE VARIABLES (SPHERICITY, C–PARM)

QCD TO ALL ORDERS

TRANSVERSE MOMENTUM RESUMMATION

SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

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Received 8 February 1979

QED

EIKONAL PHOTON EMISSION

$$\alpha^n \frac{p\cdot\epsilon_1}{k_1\cdot p}\frac{p\cdot\epsilon_2}{(k_1+k_2)\cdot p}\cdots\frac{p\cdot\epsilon_n}{(k_1+k_2\cdots+k_n)\cdot p}+(n!-1)\text{ permutations}$$

$$=\alpha^n\frac{p\cdot\epsilon_1}{k_1\cdot p}\frac{p\cdot\epsilon_2}{k_2\cdot p}\cdots\frac{p\cdot\epsilon_n}{k_n\cdot p},$$

SOFT PHOTON EXPONENTIATION

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_\perp^2}=\frac{1}{p_\perp^2}\frac{\alpha}{\pi}\ln\frac{s}{p_\perp^2}\exp\left[-\frac{\alpha}{2\pi}\ln^2\frac{s}{p_\perp^2}\right].$$

QCD

INCLUSION OF RUNNING COUPLING

$$\tilde{F}(Q^2,b)=\exp\Delta(b),$$

$$\Delta(b)\equiv\frac{1}{\pi}\int d^2k_\perp\left[\frac{4\alpha(k_\perp^2)\ln(Q^2/k_\perp^2)}{3\pi k_\perp^2}\right]_+\exp i\boldsymbol{b}\cdot\boldsymbol{k}_\perp$$

$$=\frac{1}{\pi}\int d^2k_\perp\frac{4}{3\pi}\frac{\alpha(k_\perp^2)\ln(Q^2/k_\perp^2)}{k_\perp^2}[\exp(i\boldsymbol{b}\cdot\boldsymbol{k}_\perp)-1].$$

INTUITIONS

- INTEGRATE OVER $\alpha_s(k_t)$
- b-SPACE APPROACH (FACTORIZES PHASE SPACE)

"an error...was corrected by Parisi and Petronzio...These authors introduced more powerful techniques: they worked with the Fourier transform with respect to Q_T... and they showed the usefulness of soft gluon methods" (Collins, Soper, Sterman, 1984)

QCD TO ALL ORDERS
THRESHOLD RESUMMATION

SUMMING LARGE PERTURBATIVE CORRECTIONS IN QCD

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KINEMATICS

Now, for simple kinematical reasons $(p_\perp^{\mathrm{m}})^2$ is proportional to $q^2(1-x)$ in deep inelastic scattering and to $Q^2(1-\tau)^2$ in Drell–Yan processes. The difference

p_t^m: max transverse mom.

NAIVE EXPONENTIATION

$$C_{\mathrm{dis}}(q^2, n) \approx \exp\{[\alpha(q^2)/2\pi]\,(\tfrac{4}{3}\ln^2 n - \tfrac{4}{9}\pi^2)\}\,,$$

$$C_{\mathrm{DY}}(Q^2, n) \approx \exp\{[\alpha(Q^2)/2\pi]\,(\tfrac{16}{3}\ln^2 n - \tfrac{4}{9}\pi^2)\}\,.$$

n: Mellin variable

INCLUSION OF RUNNING COUPLING

$$C_{\mathrm{dis}}(q^2, n) \approx \exp\{[\alpha(q^2)/2\pi]\,(\tfrac{4}{3}\ln^2 n - \tfrac{4}{9}\pi^2)\}\,,$$

$$C_{\mathrm{DY}}(Q^2, n) \approx \exp\{[\alpha(Q^2)/2\pi]\,(\tfrac{16}{3}\ln^2 n - \tfrac{4}{9}\pi^2)\}\,.$$

TIMELIKE VS. SPACELIKE

$$\lim_{Q^2\to\infty} |F(Q^2)/F(-Q^2)|^2 = \exp\{\tfrac{4}{3}\,[\alpha(Q^2)/2\pi]\,\pi^2\}.$$

"The exponentiation of the factor $\frac{4}{9}\pi^2$ is rather doubtful"

INTUITIONS

- KINEMATIC ORIGIN OF SOFT LOGS
- MELLIN-SPACE EXPONENTIATION
- RUNNING $\alpha_s(Q^2/n)$
- THE ORIGIN OF LARGE NLO π^2; PROVEN BY MAGNEA AND STERMAN (1990), "DISCOVERED" BY AHRENS, BECHER, NEUBERT, YANG IN 2009

CONCLUSION

QCD TO ALL ORDERS

THE HIGH-ORDER BEHAVIOR OF THE PERTURBATIVE EXPANSION

THE PHYSICAL BASIS OF THE ASYMPTOTIC ESTIMATES

IN PERTURBATION THEORY +

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FRANCE

+ Lectures given at the 1977 CARGESE Summer Institute (FRANCE)

- DIVERGENCE OF PERTURBATIVE EXPANSION IN QFT
- SINGULARITIES IN BOREL TRANSFORM: RENORMALONS, INSTANTONS, LANDAU CUT

The reader may think that in spite of our efforts, the situation is still a mess. That is true. However, in these last few years a progress has been made : we do not know yet how to get correct answers for a field theory of strong interactions, but we begin to understand which are the right questions to ask.