



THE DETERMINATION OF α_s : **STATUS AND PROSPECTS**

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WHY α_s ?

THE PDG VALUE

$\alpha_s(M_z) = 0.1181 \pm 0.0011$

(PDG 2016 rev.: Bethke, Dissertori, Salam)

- $\alpha_s \sim \frac{1}{10}$ at the Higgs mass: expansion parameter quite large
- $\Delta_{\alpha} \sim 1\% \Rightarrow$ UNCERTAINTY ON PROCESS WHICH STARTS AT, SAY $O(\alpha_s^2)$ (HIGGS IN GLUON FUSION) IS ~ 2% OF LO+ 3% OF NLO + ...

THE GOOD NEWS



"The agreement among so many different ways of measuring α_s is a strong quantitative test of QCD"

(G. Altarelli, "The QCD running coupling and its measurement", 2013)

THE BAD NEWS AN EXAMPLE: HIGGS IN GLUON FUSION

- MAIN DISCOVERY CHANNEL; SENSITIVE BSM PROBE (TOP YUKAWA)
- STARTS AT $O(\alpha_s^2)$, 100% NLO CORRECTION
- DRIVEN BY THE GLUON PDF
- ONLY HADRON COLLIDER PROCESS KNOWN UP TO N^3LO (Anastasiou et al., 2016)



% UNCERTAINTIES ON TOTAL CROSS-SECTION

- MISSING HIGHER ORDER & PDF UNCERTAINTIES CUT BY FACTOR 2 BETWEEN HIGGS DISCOVERY (YR1, 2011) & LHC RUN II (YR4, 2016)
- α_s UNCERTAINTY UNCHANGED

QCD PROGRESS I

NNLO QCD RESULTS



BETWEEN HIGGS DISCOVERY & LHC RUN II

- PERTURBATIVE STANDARD: FROM NLO TO NNLO (& BEYOND)
- PDF standard: uncertainties & accuracy from $6 \div 7\%$ to $\lesssim 3\%$

QCD PROGRESS II: PERTURBATIVE ACCURACY & PDFs

- 2000
 - NLO QCD EVOLUTION, MOST PROCESSES KNOWN AT NLO
 - PDF UNCERTAINTIES UNKNOWN
 - data-theory comparison $\sim 10-20\%$
- 2010
 - NNLO QCD EVOLUTION, INCLUSIVE STANDARD CANDLES KNOWN AT NNLO
 - PDF uncertainties $\sim 10\%$, accuracy?
 - DATA-THEORY COMPARISON $\sim 5-10\%$
- 2017
 - NNLO QCD PREDICTIONS INCLUDING FULLY EXCLUSIVE FINAL STATES
 - -~ PDF uncertainties $\lesssim 5\%$ with reliable accuracy

- data-theory comparison $\sim 1-5\%$



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 α_s : HOW?

THE PDG AVERAGE WHERE DOES IT COME FROM?



- PDG AVERAGE: χ^2 -AVERAGING OF SIX PRE-AVERAGES, CHOSEN TO BE MAXIMALLY UNCORRELATED
- χ^2 AVERAGING \Rightarrow UNCERTAINTIES INFLATED OR CORRELATED UNTIL $\chi^2/dof = 1$
- EACH PRE-AVERAGE IS THE SIMPLE (UNWEIGHTED) AVERAGE OF ITS COMPONENTS, UNCERTAINTY ↔ AVERAGE OF UNCERTAINTIES (OR STANDARD DEVIATION OF VALUES IF LARGER)
- ONLY PUBLISHED NNLO DETERMINATIONS INCLUDED
- GLOBAL ELECTROWEAK FIT (NO PRE-AVERAGING)
- ONLY ONE LHC DETERMINATION: TOP (NO PRE-AVERAGING)

CASE STUDY I: STRUCTURE FUNCTIONS

- "STRUCTURE FUNCTION": α_s determined SIMULTANEOUSLY WITH PDFS IN A MORE OR LESS GLOBAL FIT
- BIG SPREAD OF VALUES, SOME VERY CONTROVERSIAL



BASIC PRINCIPLE: SCALING VIOLATIONS



• $\alpha_s \& \text{PDFs}$ (GLUON) STRONGLY CORRELATED $\frac{d}{d \ln Q^2} F_2(x, Q^2) \sim \alpha_s(Q^2) F_2(x, Q^2) + \alpha_s(Q^2) G(x, Q^2)$

MANY SOURCES OF UNCERTAINTY :

- PDF parametrization & propagation of PDF uncertainties
- TREATMENT OF EXPERIMENTAL SYSTEMATICS IN THE GLOBAL FIT
- HEAVY QUARK MASSES: FFN vs. VFN schemes; PERTURTBATIVE vs. FITTED CHARM; VALUES OF HQ MASSES
- BBG: DRIVEN DOWNWARDS BY NEGLECT OF LARGE NONSINGLET CONTRIBUTION (Martin, 2011)
- ABM: DRIVEN DOWNWARDS BY FIXED-FLAVOR NUMBER TREATMENT OF HEAVY QUARKS (Thorne, 2014)
- JR: BASED ON OBSOLETE PDF FIT
- NNPDF, MMHT: NO SERIOUS ESTIMATE OF THEORY UNCERTAINTIES

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(CELLO collab., 1987)

- MULTIPLICATIVE UNCERTAINTIES IN COVARIANCE MATRIX
 ⇒ FIT BIASED DOWNWARDS IF DATA INCONSISTENT (d'Agostini, 1994)
 EQUIVALENT TO RESCALING DATA BUT NOT UNCERTAINTIES
- MUST USE ITERATIVE PROCEDURE COVARIANCE MATRIX COMPUTED FROM PREVIOUS FIT (NNPDF, 2010)



- χ^2 COMPUTED FROM COVARIANCE MATRIX \Rightarrow **BIASED** LOW FIT FAVORED
- LESS EVOLUTION \Leftrightarrow LOW α_s
- ONLY WHEN MULTIPLICATIVE UNCERTAINTIES DOMINATE COLLIDER ONLY, NOT FIXED TARGET

CASE STUDY II: τ DECAYS

- LEPTONIC INTIAL STATE: NO DEPENDENCE ON PDFS
- LOW SCALE ($m_{\tau} = 1.78$ GeV):
 - UNCERTAINTY AT HIGH SCALE M_z SHRINKS
 - LARGE PERTURBATIVE & NONPERTURBATIVE CORRECTIONS
- SPREAD OF VALUES DUE TO NONPERTURBATIVE TREATMENT



BASIC PRINCIPLE: HADRONIC DECAY WIDTH $R_{\tau} = \frac{\Gamma(\tau^{-} \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma(\tau^{-} \rightarrow \nu_{\tau} e^{-} \bar{\nu}_{e})}$ $= f(\alpha_{s}(m_{\tau}))$

- HADRONIC WIDTH $\Leftrightarrow W$ SELF-ENERGY $\Pi(q^2)$ $\Pi^{\mu\nu}(q) = i \int d^4 x e^{iqx} \langle T\left(J_{\mu}(x) J_{\nu}^{\dagger}(0)\right) \rangle$
- EXPRESSED USING ANALYTICITY IN TERMS OF INTEGRAL OVER $\Pi(s)$ in complex s plane on the circle $|s|=m_\tau^2$
- LEADING ORDER WILSON COEFFN. PERTURBATIVELY KNOWN UP TO $O(\alpha_s^4)$ (Baikov, Chetyrkin, Kühn, 2008)



•
$$R_{\tau} = \frac{\Gamma(\tau^- \to \nu_{\tau} + \text{hadrons})}{\Gamma(\tau^- \to \nu_{\tau} e^- \bar{\nu}_e)}$$

• $\Pi^{\mu\nu}(q) = i \int d^4x e^{iqx} \langle T\left(J_{\mu}(x)J_{\nu}^{\dagger}(0)\right) \rangle$



PERTURBATIVE UNCERTAINTIES

- EXPANSION IN α_s OF CONTOUR INTEGRAL (FOPT) VS EXPANSION OF THE INTEGRAND (CIPT) \Rightarrow FOPT IS A NON-CONVERGENT SERIES, BUT CIPT MAY CONTAIN RENORMALONS
- FIRST SUBLEADING OPERATOR FOR $\Pi(q^2)$ is dimension $4 \propto \frac{\langle O_4 \rangle}{m_{\tau}^4}$ RENORMALON-INDUCED CONTRIBUTION $\propto \frac{\Lambda^2}{m_{\tau}^2}$?
- CIPT AND FOPT AVERAGED, \Rightarrow DIFFERENCE IN $\alpha_s(M_z)$ OF ORDER OF $\sim 1.5 - 2\%$

NONPERTURBATIVE UNCERTAINTIES



- MOMENTS OF THE DECAY INVARIANT-MASS DISTRIBUTION ⇔ WEIGHTED INTEGRALS OF THE SPECTRAL FUNCTION
- NP CORRECTIONS EXTRACTED BY COMPARISON
- **RESONANCE CONTRIBUTIONS** TO INTEGRAL: SUPPRESSED BY CHOICE OF WEIGHT OR MODELED?
- \Rightarrow DIFFERENCE IN $\alpha_s(M_z)$ OF ORDER OF $\sim 3\%$

CASE STUDY III: e^+e^- JETS & SHAPES

- LEP (& PETRA) JETS: LEPTONIC INITIAL STATE, WIDE SET OF DATA AND OBSERVABLES
- FINAL STATE HADRONS \Rightarrow NONPERTURBATIVE PHYSICS
- OUTLIER DETERMINATION, VERY SMALL UNCERTAINTY



BASIC PRINCIPLE: JET RATES & SHAPE THRUST: $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$

$$C = \frac{\frac{\text{C-PARAMETER:}}{3}}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

broadening, jet mass, two-to-three jet transition; three-jet rate

- NNLO+NLL PERTURBATIVE COMPUTATION MATCHED TO MONTE CARLO HADRONIZATION:
 - SIMULTANEOUS FIT TO SIX EVENT SHAPES: ALEPH, OPAL (LEP), JADE (PETRA)
 - THREE-JET RATES: ALEPH (NNLO ONLY), JADE
- NNLO+N^kLL MATCHED TO NONPERTURBATIVE MODELING:
 - NNLO+NLL EFFECTIVE COUPLING (THRUST)
 - NNLO+NNLL DISPERSIVE MODEL (THRUST)
 - NNLO+N³LL (PADÉ) SCET FITTED SHAPE FUNCTION (THRUST & C-PARAMETER)

CASE STUDY III: e^+e^- JETS & SHAPES UNCERTAINTIES (CONSERVATIVE): ALEPH EVENT SHAPES

- STAT. (BINNED χ^2 FIT) & EXP. UNCERTAINTIES (VARY CUTS) SMALL
- PERTURBATIVE AND NONPERTURBATIVE UNCERTAINTIES DOMINANT





PERTURBATIVE UNCERTAINTIES

- RESUMMATION NEEDED FOR ACCURATE PREDICTIONS IN WIDE RANGE ⇒ REDUCED DISPERSION OF RESULTS
- INCOMPLETE SCALE CANCELLATION WHEN NNLO MATCHED TO NLL (NEED NNLL)
 ⇒ INCREASED UNCERTAINTY
- $\Delta \alpha_s(M_z) \sim 3.5\%$

NONPERTURBATIVE UNCERTAINTIES

MC VARIATION VS. EXP UNC.

- MC REANALYSIS USING MODERN PS (MC@NLO & POWEG)
 ⇒ DISCREPANCY (TUNING ISSUES?)
- PYTHIA & HERWIG HADRONIZATION: SOME OBSERVABLES (THRUST, C PARM.) \Rightarrow UNDERESTIMATED HADR. CORRECTIONS?
- $\Delta \alpha_s(M_z) \gtrsim 1\%$



(Dissertori et al., 2009)

CASE STUDY III: e^+e^- JETS & SHAPES UNCERTAINTIES (AGGRESSIVE): THRUST FROM SCET

- GLOBAL FIT TO VERY LARGE NUMBER OF THRUST DATA (AMY@KEK; TASSO, JADE@PETRA; SLC; ALEPH, OPAL, DELPI, L3 @ LEP)
- PERMILLE EXPERIMENTAL, HADRONIZATION & PERTURBATIVE UNCERTAINTIES





PERTURBATIVE UNCERTAINTIES

- RESUMMATION (N³LL PADÉ), NONPERTURBATIVE FUNCTION & RENORMALON SUBTRACTION ADDED TO FIXED ORDER
- "THEORY SCAN" UNCERTAINTY: TWO-PARAMETER α_s , Ω_1 FIT WITH 12 THEORY PARMS VARIED \Rightarrow CONSTANT DECREASE OF SPREAD
- $\Delta \alpha_s(M_z) \lesssim 1\%$

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- GLOBAL FIT TO VERY LARGE NUMBER OF THRUST DATA (AMY@KEK; TASSO, JADE@PETRA; SLC; ALEPH, OPAL, DELPI, L3 @ LEP)
- PERMILLE EXPT., HADR. & PERT. UNCERTAINTIES





NONPERTURBATIVE UNCERTAINTIES

- ONE-SIGMA ELLIPSE OF TWO-PARAMETER FIT \Rightarrow HADRONIZATION UNCERTAINTY
- LARGE STATISTICS, VERY SMALL UNCERTAINTY: $\Delta \alpha_s(M_z) \sim 0.5\%$
- WHY IS THE CENTRAL VALUE SO LOW?

(Abbate et al., 2010)

• DISPERSIVE MODEL+STANDARD QCD LEADS TO $\Delta \alpha_s(M_z) \sim 3\%$ (only ALEPH, L3, TASSO) NO RENORMALONS, NO N³LL PADÉ)

(Gehrmann, Luisoni, Monni et al., 2012)

• ARE RESULTS STABLE UPON REBINNING?

"I think that this is a good example of an underestimated error which is obtained within a given machinery without considering the limits of the method itself"

(G.Altarelli, 2013)



LESSONS LEARNT

- COMPLEX ANALYSIS LEADS TO MANY SOURCES OF UNCERTAINTY, SOME UNEXPECTED
- NONPERTURBATIVE & PDF UNCERTAINTIES HARD TO ESTIMATE
- AGGRESSIVE ERROR ESTIMATE SIGNALED BY WIDE DISPERSION OF RESULTS

WHICH α_s ?

THE MEANING OF THE α_s UNCERTAINTY

TWO DETERMINATIONS IN PDG NOT PRE-AVERAGED: GLOBAL ELECTROWEAK FIT & TOP-PAIR TOTAL CROSS SECTION @ LHC (ONLY HADRON COLLIDER DETERMINATION IN PDG AVERAGE)



- DO THEY HAVE THE SAME UNCERTAINTY?
 - EASY (CONSERVATIVE?) UNCERTAINTY ESTIMATE: THE GLOBAL ELECTROWEAK FIT
 - MUST ASSUME STANDARD MODEL
 - FIT HIGHLY CONSTRAINED & CONSISTENT (14 d.o.f., $\chi^2 = 17.8$) (ALL ANOMALIES GONE)
 - GOOD CONSISTENCY OF VALUES FROM GLOBAL FIT (α_s ONE OF THE FIT PARAMETERS) & PREDICTION (α_s NOT A FIT PARAMETER)
 - CONSTANT IMPROVEMENT
 - NOW HIGGS MASS ADDED TO FIT







THE MEANING OF THE α_s UNCERTAINTY

• TWO DETERMINATIONS IN PDG NOT PRE-AVERAGED: GLOBAL ELECTROWEAK FIT & TOP-PAIR TOTAL CROSS SECTION @ LHC (ONLY HADRON COLLIDER DETERMINATION IN PDG AVERAGE)



• DO THEY HAVE THE SAME UNCERTAINTY?

DIFFICULT (AGGRESSIVE?) UNCERTAINTY ESTIMATE: TOP PAIR PRODUCTION AT THE LHC

- INCLUSIVE "STANDARD CANDLE", CROSS-SECTION KNOWN TO NNLO+NNLL
- DEPENDS ON α_s & PDFs
- BAYESIAN UPDATE OF PRIOR DISTRIBUTION $f(\sigma_{t\bar{t}})$ \Rightarrow JOINT POSTERIOR FOR $\sigma_{t\bar{t}}$, m_t & α_s FOLDING WITH PROBABILITIES FOR PDF & THEORY UNCERTAINTY





- DO WE FULLY CONTROL THE **PDF** UNCERTAINTY?
- DOES THE LOW α_s VALUE REFLECT AN OVERESTIMATED GLUON PDF?



GRESHAM'S LAW OF α_s **DETERMINATION** "Bad determinations drive our good determinations"



(T. Gresham, 1519-1579)

- ACCURATELY ASSESSED UNCERTAINTY \Rightarrow LARGER!
- DETERMINATIONS WITH LARGE UNCERTAINTY ARE DISFAVORED W.R. TO SMALL UNCERTAINTY

"CONSERVATIVE" α_s ?

"In my opinion one should select few theoretically simplest processes for measuring α_s and consider all other ways as tests of the theory."

(G.Altarelli, 2013)



• TAKE, FOR SAKE OF ARGUMENT, PDF-INDEPENDENT DETERMINATIONS, WITH CONSERVATIVE UNCERTAINTIES:

 $-\tau$ DECAYS $\Rightarrow \alpha_s(M_z) = 0.119 \pm 0.002$

- EW FIT (HEPfit) $\Rightarrow \alpha_s(M_z) = 0.1185 \pm 0.0028$
- COMBINED RESULT: $\alpha_s(M_z) = 0.1188 \pm 0.0016$

OUTLOOK

- SURPRISING LACK OF PROGRESS:
 - NO SINGLE RELIABLE AND ACCURATE DETERMINATION
 - OUTLIERS NOT FULLY UNDERSTOOD
- FUTURE IMPROVEMENTS:
 - − LHC-RUN II & HL-LHC DATA \Rightarrow GLOBAL EW FIT, PDFs
 - LATTICE AS AN INDEPENDENT HANDLE \Rightarrow OUTLIERS DISCARDED

there is nothing new under the sun but there are lots of old things we don't know

Ambrose Bierce



THE RUNNING OF α_s



(PDG, 2016)

- CONSISTENT SCALE DEPENDENCE OVER THREE ORDERS OF MAGNITUDE
- RUNNING KNOWN TO FIVE LOOPS (Baikov, Chetyrkin, Kühn, 2016)



- METHODOLOGY: INDEPENDENTLY PARAMETRIZED CHARM PDF
- LHC DATA : TOP RAPIDIDITY, $Z p_T$, LHCB & ATLAS W & Z PRODUCTION



- FITTING CHARM ENHANCES GLUON IN GGF RANGE BY SEVERAL PERCENT
- NEW DATA (ESPECIALLY LHCB & ATLAS $w \ z \ 2011$) ENHANCE QUARKS IN VALENCE REGION